GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM
REACCREDITED WITH NAAC 'A' GRADE

B.Sc (Regular) Semester-V
(2022-23 Admitted Batch)

Paper	: Mathematical Special Functions	Time : 3 Hrs
Marks	: 60	Date : 29-11-2024

Section A

Answer *five* questions. Chose one question from each Unit. All questions carry equal marks [$5 \times 8M = 40M$]

Unit - I

1. a) Prove that $B(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

   (or)

   b) Prove that $\int_0^{\frac{\pi}{2}} \sin^{2p-1} x \cos^{2q-1} x\, dx = \frac{\Gamma p \Gamma q}{2\Gamma(p+q)}$.

Unit - II

2. a) Find seric series solution for the differential equation $x^2 y'' + xy' + (v^2 - n^2)y = 0$ using Frobinius method.

   (or)

   b) Find two independent solutions for the differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ using power series method.

Unit - III

3. a) State and prove Rodrigue's formula for Hermite polynomials.

   (or)

   b) Show that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ and $H_{2n+1}(0) = 0$

Unit - IV

4. a) Prove that

$$\int_{-1}^1 P_m(x)P_n(x)\,dx = \begin{cases} 0, & \text{if } m \neq n \\ \frac{2}{2n+1}, & \text{if } m = n \end{cases}$$

   (or)

   b) Show that $P_n(x) = \frac{1}{n!2^n} \frac{d^n}{dx^n}(x^2 - 1)^2$

Unit - V

5. a) Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

   (or)

   b) Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ and $xJ_n'(x) = xJ_{n-1}(x) - nJ_n(x)$

Section B

Answer any *five* questions [5 ×4 $M = 20M$]

6. Evaluate $B(4,5)$.

7. Prove that $\Gamma n = n\Gamma(n-1)$

8. Find series solution of $y' + y = 0$

9. Clasify $x = -1$, $x = 0$ and $x = 1$ for the differential equation $x^3(x+1)y'' + xy' + y = 0$ in terms of ordinary point, regular singular point and irregular singular point.

10. Prove that $H_n'(x) = 2nH_{n-1}(x)$.

11. Prove that $H_3(x) = 8x^3 - 12x$.

12. Show that $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$.

13. Show that $x^2 = \frac{1}{3}P_0(x) + \frac{2}{3}P_2(x)$.

14. Show that $J_n(-x) = (-1)^n J_n(x)$.

15. Write series form expression for $J_0(x)$.