

**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-I**

**Credits-4**

**Hrs/Wk-5**

**Course No-1 & Course Title- Differential Equations**

**it-1-Differential Equations of first order and first degree**

Linear Differential Equations – Bernoulli's Equations - Exact Differential Equations –Integrating factors - Equations reducible to Exact Equations by Integrating Factors -

i) Inspection Method    ii)  $\frac{1}{Mx+Ny}$                        $\frac{1}{Mx-N}$

**UNIT-2-Differential Equations of first order but not of first degree**

Equations solvable for  $p$ , Equations solvable for  $y$ , Equations solvable for  $x$  –Clairaut's equation

**UNIT-3-Higher order linear differential equations**

Solutions of homogeneous linear differential equations of order  $n$  with constant coefficients -  
Solutions of non-homogeneous linear differential equations with constant coefficients by means of polynomial operators

(i) P.I. of  $(D) = Q$  when  $Q(x)=e^{ax}$

(ii) P.I. of  $(D) = Q$  when  $Q(x)=\sin bx$  or  $\cos bx$

**UNIT-4-Higher order linear differential equations (continued.)**

Solution to a non-homogeneous linear differential equation with constant coefficients

P.I. of  $(D) = Q$  when  $Q = bx^k$

P.I. of  $(D) = Q$  when  $Q = e^{ax}V$ , where  $V$  is a function of  $x$

**UNIT-5-Higher order linear differential equations with non-constant coefficients**

Solution to a non-homogeneous linear differential equation with constant coefficients P.I. of  $(D) = Q$  when

$Q = xV$ , where  $V$  is a function of  $x$ .

Linear differential Equations with non-constant coefficients; Method of variation of parameters

### **Additng:**

Unit-1-  $\frac{1}{M} \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$  and  $\frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$  methods.

Unit-2- Equations that do not contain x(or y)- Orthogonal Trajectories: Cartesian and Polar forms.

Unit-3-  $f(D)y = Q$  when Q is a function of x.

Unit-5- P.I. of  $f(D)y = Q$  when  $Q=x^m V$ , where V is a function of x-Linear differential Equations with non-constant coefficients; Cauchy-Euler Equation; LegendreEquation.

### **Activities**

The activities planned throughout the Differential Equations course include variety of interactive and evaluative methods such as quizzes, assignments, seminars, and student presentations. Students will also engage in a mini project, prepare concept flowcharts, and participate in operator method chart activities. Peer teaching sessions, LMS-based online quizzes, and board work challenges will foster collaborative and digital learning. Additionally, poster presentations on applications and visual aids like chalk talks will be incorporated to support diverse learning styles and deepen conceptual clarity.

### **Text Book**

Differential Equations and Their Applications by Zafar Ahsan, published by Prentice-Hall of India Pvt. Ltd, New Delhi-Second edition.

### **Reference Books**

1. Ordinary and Partial Differential Equations by Dr. M.D. Raisinghania, published by S. Chand & Company, New Delhi.
2. Differential Equations with applications and programs – S. Balachandra Rao & HR Anuradha- Universities Press.
3. Differential Equations -Srinivas Vangala&Madhu Rajesh, published by Spectrum University Press

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

I B.Sc. HONOURS MATHEMATICS (MAJOR)-SEMESTER-I

TIME:3Hrs.

COURSE-1, DIFFERENTIAL EQUATIONS

MARKS: 60 M

SECTION-A

Answer ALL FIVE Questions Each question carries EIGHT marks.

5 X 8M = 40M.

1) a) Solve  $\frac{dy}{dx}(x^2y^3 + xy) = 1$

(OR)

b) Solve  $(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0$

2) a) Solve  $p^2 + 2p \cot x = y^2$

(OR)

b) Solve  $2px = 2\tan y + p^3 \cos^2 y$

3) a) Solve  $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$

(OR)

b) Solve  $(D^2+4)y = e^x + \sin 2x + \cos 2x$

4) a) Solve  $(D^2-6D+13)y = 8e^{3x} \sin 2x$

(OR)

b) Solve  $(D^2-4D+3)y = 2xe^{3x} + 3e^x \cos 2x$

5) a) Solve  $(D^2+1)y = \operatorname{cosec} x$  by using the method of variation of parameters

(OR)

b) Solve  $(D^4+2D^2+1)y = x^2 \cos x$

SECTION -B

Answer any FIVE of the following .Each question carries FOUR marks.

5 x 4M = 20M

6. Solve  $(x^2 + 1) \frac{dy}{dx} + 4xy = 1/x^2 + 1$

7. Solve  $\cos^2 x \frac{dy}{dx} + y = \tan x$

8. Solve  $y = 2xp + x^2p^4$

9. Solve  $(py+x)(px-y) = 2p$

10. Solve  $(D^4-4D^3+6D^2-4D+1)y = 0$

11. Solve  $(D^2+4)y = \sin 2X$

12. Solve  $(D^2 - 2D + 1)y = x2e^{3x}$

13. Solve  $(D^2-4)y = x \sinh x$

14. Solve  $\frac{d^2y}{dx^2} + (x-2)\frac{dy}{dx} - 2y = x^3$

15. Solve  $(D^2 + 4)y = x \sin x$ .

# **GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-I**

**Credits-4**

**Hrs/Wk-5**

## **Course No-2 & Course Title- ANALYTICAL SOLID GEOMETRY**

### Course Objectives

1. To introduce fundamental concepts of planes, lines, and spheres in 3D geometry.
2. To develop analytical skills for deriving equations of planes, lines, and spheres in different forms.
3. To analyze geometric relationships, including angles, distances, and intersections between lines, planes, and spheres.
4. To study advanced properties of spheres, such as tangents, polar planes, and orthogonality conditions.
5. To apply geometric principles to solve problems involving co-planarity, shortest distances, and sphere-line/plane interactions.

### **Unit-1-The Plane**

Equation of plane in terms of its intercepts on the axis - Equations of the plane through the given points - Length of the perpendicular from a given point to a given plane- Bisectors of angles between two planes - Combined equation of two planes.

### **Unit-2-The Line**

Equation of a line in various forms- Angle between a line and a plane - The condition that a given line may lie in a given plane - The condition that two given lines are coplanar - Number of arbitrary constants in the equations of straight line - Sets of conditions which determine a line

### **Unit-3-The Line (continued)**

- The shortest distance between two lines - The length and equations of the line of shortest distance between two straight lines - Length of the perpendicular from a given point to a given line.

### **Unit-4-The Sphere**

Definition and equation of the sphere - Equation of the sphere through four given points - Plane sections of a sphere - Intersection of two spheres - Equation of a circle - Sphere through a given circle -Intersection of a sphere and a line .

### **Unit-5- Spheres (continued)**

- Power of a point - Tangent plane - Plane of contact; Polar plane-Pole of a Plane - Conjugate points - Conjugate planes-Angle of intersection of two spheres - Conditions for two spheres to be orthogonal - Radical plane- Coaxial system of spheres -**Limiting Point**.

#### **Adding:**

Unit-1- Angle between two Planes-Two sides of a Plane-Distance between parallel planes -Orthogonal projection on a plane.

Unit-3- Area of a triangle-Intersection of three planes.

Unit-5--Touching spheres -Radical line- Radical centre- Simplified form of the equation of two spheres.

#### **Activities:**

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

#### **Text Book:**

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand & Company Ltd. 7th Edition.

#### **Reference Books:**

1. A text Book of Analytical Geometry of Three Dimensions, by P.K.Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.
2. Co-ordinate Geometry of two and three dimensions by P.Balasubrahmanyam, K.Y.Subrahmanyam, G.R.Venkataraman published by Tata McGraw-Hill Publishers.
3. Solid Geometry by B. Rama Bhupal Reddy, published by Spectrum University Press.

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**GOVERNMENT COLLEGE FOR WOMEN,(AUTONOMOUS),SRIKAKULAM**

**I B.Sc. HONOURS MATHEMATICS (MAJOR)-SEMESTER-I**

**TIME: 3Hrs**

**ANALYTICAL SOLID GEOMETRY**

**Max.Marks:60Marks**

**SECTION – A**

**Answer ALL questions ,each question carries eight marks.**

**5x8M=40Marks**

- 1.) a) Find the equation of the plane through the points (2,2,1) (9,3,6) and perpendicular to the plane  $2x+6y+6z=9$ .

(OR)

- b) A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of the tetrahedron OABC is  $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$ .
- 2) a) Find the image of the point (1,3,4) in the plane  $2xy+z+3=0$ .

(OR)

- b) Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  ;  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  are coplanar. Also find their point of intersection and the plane containing the lines.
- 3) a) Show that the two circles  $x^2+y^2+z^2-y+2z=0$ ,  $x-y+z=2$ ;  $x^2+y^2+z^2+x-3y+z-5=0$ ,  $2x-y+4z-1=0$  lie on the same sphere and find its equation.

(OR)

- b) Find the equation of the sphere which touches the plane  $3x+2y-z+2=0$  at (1,-2,1) and cuts orthogonally the sphere  $x^2+y^2+z^2-4x+6y$
- 4 ) a) Find the limiting points of the coaxial system of spheres

$$x^2+y^2+z^2+8x+2y-2z+32=0, x^2+y^2+z^2-7x+z+23=0.$$

(OR)

- b) Show that the plane  $2x-2y+z+12=0$  touches the sphere

$$x^2+y^2+z^2-2x-4y+2z-3=0 \text{ and find the point of contact.}$$

5) a) Prove that the equation  $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$  represents a cone that touches the coordinate planes and find its reciprocal cone.

(OR )

b) Find the equation of the sphere  $x^2+y^2+z^2-2x+4y-1=0$  having its generators parallel to the line  $x=y=z$ .

### SECTION-B

ANSWER ANY FIVE OF THE FOLLOWING.

5x4M=20Mark

6) Find the bisector of the angle between the planes  $3x-6y+2z+5=0$ ,  $4x-12y+3z-3=0$  which contains the origin .

7) Show that the equation  $2x^2-6y^2-12z^2+18yz+2zx+xy=0$  represents a pair of planes. Also find the angle between them.

8) Find the image of the point  $(2,-1,3)$  in the plane  $3x-2y-z=9$ .

9) Find the shortest distance and the equations of S.D between the lines  $3x-9y+5z=0=x+y-z$ ;  $6x+8y+3z-10=0=x+2y+z-3$ .

10) Show that the four points  $(-8,5,2)$   $(-5,2,2)$   $(-7,6,6)$   $(-4,3,6)$  are concyclic.

11) Show that the plane  $2x-2y+z+12=0$  touches the sphere  $x^2+y^2+z^2-2x-4y+2z-3=0$  and find the point of Contact.

12) Find the equation of the sphere which touches the plane  $3x+2y-z+2=0$  at  $(1,-2,1)$  and cuts Orthogonally the sphere  $x^2+y^2+z^2-4x+6y+4=0$ .

13) Find the equation of the sphere of the coaxial system  $x^2+y^2+z^2-5+\lambda (2x+y+3z-3)=0$  which touch the plane  $3x+4y-15=0$ .

14) Find the equation of the cone whose vertex is  $(1,1,0)$  and whose guiding curve is  $y=0, x^2+z^2=4$ .

15) Find the equation to the right circular cone whose vertex is  $P(2,-3,5)$  axis PQ which makes equal angles with the axes and which passes through  $A(1,-2,3)$ .

# **GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-I I**

**Credits-4**

**Hrs/Wk-5**

## **Course No-3 & Course Title- GROUP THEORY**

### Course Objectives

1. To introduce students to the foundational concepts of algebraic structures with a focus on groups.
2. To develop an understanding of subgroups, cosets, and their relevance in group theory.
3. To explore the properties and significance of normal subgroups and their role in constructing quotient groups.
4. To study and apply the concepts of group homomorphisms, isomorphisms, and the fundamental theorem of homomorphism.
5. To examine the structure and properties of permutation and cyclic groups, including their role in group classification.

### **Unit – 1-Groups**

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

### **Unit – 2 Sub Groups**

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition-examples-criterion for a complex to be a subgroups; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – properties of Cosets – Index of a subgroups of a finite groups – Lagrange's Theorem.

### **Unit – 3 Normal Subgroups**

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups Sub group of index 2 is a normal sub group

### **Unit – 4 Homomorphisms**

Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

## **Unit – 5 Permutations and Cyclic Groups**

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

### **Adding:**

Unit-1- Order of an element of a group

Unit-2- Normalizer of an element of a group-Self-conjugate element of a group.

Unit-3-Simple group.

### **Activities:**

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

### **Text Book:**

Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, Krishna Prakashan Media Pvt. Ltd., Meerut.

### **Reference Books:**

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

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**GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM**

**I B.Sc. HONOURS, MATHEMATICS (MAJOR), SEMESTER-II**

**TIME: 3Hrs**

**COURSE-3, GROUP THEORY**

**Max.Marks:60**

**SECTION-A**

**Answer ALL FIVE Questions. Each question carries EIGHT marks.**

**5x8M=40M.**

- 1) a) Show that the set  $Q^+$  of all positive rational numbers forms an abelian group under the

Composition defined by ' $\circ$ ' such that  $a \circ b = \frac{ab}{3}$  for  $a, b \in Q^+$ .

(OR)

- b) Prove that the set of  $n^{th}$  roots of unity under multiplication forms a finite abelian group.

- 2) a) If H and K are two subgroups of a group G then Prove that HK is a subgroup of G iff HK=KH.

(OR)

- b) State and prove Lagrange's Theorem.

- 3) a) Prove that a subgroup H of a group G is a Normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(OR)

- b) If H is a subgroup of G and N is normal subgroup of G, Then i) H intersection N is a normal subgroup ii) N is normal subgroup of HN.

- 4) a) State and prove fundamental theorem of Homomorphism of Groups.

(Or)

- b) If f is homomorphism of a group G then the Kernel of f is a normal subgroup of G.

- 5) a) State and prove Cayley's Theorem.

(OR)

- b) If P is a prime number then every group of order P is a Cyclic Group.

**SECTION-B**

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

- 6). Show that in a Group G, for  $a, b \in G$ ,  $(ab)^2 = a^2b^2$  iff G is abelian.

- 7). Prove that Cancellation laws hold in a group.

- 8). If  $H_1$  and  $H_2$  are two Subgroups of a Group G then  $H_1 \cap H_2$  is also a Subgroup of G.

- 9). Any two right cosets of a Subgroup are either disjoint or identical

- 10). Prove that a Subgroup H of a Group G is Normal , iff  $xHx^{-1} = H, \forall x \in G$ .

- 11). The intersection of any two normal subgroups of a group is a normal subgroup.

- 12). If f is a homomorphism from a group G into a group G' then Ker f is a normal subgroup of G.

- 13).The homomorphic image of a group is a group.

- 14). Examine whether the following Permutations is Even (or) Odd.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$ .

- 15). If G is an infinite cyclic group then G has exactly two generators.

**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-I I**

**Credits-4**

**Hrs/Wk-5**

**Course No-4 & Course Title- ELEMENTARY REAL ANALYSIS**

**Course Objectives:**

1. To develop a strong foundation in the real number system and its axiomatic structure.
2. To introduce the concepts of order, bounds, completeness, and related foundational properties of real numbers.
3. To explore the properties of sets in real analysis, including neighborhoods, limit points, open and closed sets.
4. To build analytical skills in handling sequences, convergence criteria, and monotonicity.
5. To understand the behavior of in finite series and apply standard convergence tests effectively.

**Unit – 1**

Real number system - Field axioms–Properties of real numbers - Order axioms – Properties of Order relation -Principle of induction - Extended real number system – Modulus of a real number – Properties of modulus– Triangle property -Aggregates – Finite and infinite aggregates – Boundedness of an aggregate – Least upper bound (supremum) and greatest lower bound (infimum) of an aggregate – Properties of boundedness– Completeness axiom – Dedekind’s theorem- Theorem on Dedekind’s axiom and completeness axiom.

**Unit – 2**

Archimedean Property- It’s corollaries– Integral part of a real number –Denseness of the real number system – Intervals – Neighbourhood of a point -Limit point of an aggregate – Derived Set-Bolzano - Weierstrass theorem – Interior point of a set - Open and closed Sets – It’s properties (without proofs)- Countable and uncountable sets -Properties of countable sets.

**Unit –3**

Sequences – Operations of sequences -Subsequences - Range and Boundedness of Sequences - Limit of a sequence and Convergent sequence – Divergent sequence – Uniqueness of a limit – Sandwich theorem on sequences-Monotone sequences - Problems

## **Unit – 4**

Limit Point of a Sequence-Bolzano-Weierstrass theorem on subsequences–Cauchy Sequences – Cauchy’s general principle of convergence – Problems

## **Unit – 5**

Infinite Series – Convergence and divergence of series - Cauchy’s general principle of convergence for series – Series of non-negative terms -Convergence of geometric series – p series test - comparison test –D’Alembert’s ratio test– Cauchy’s  $n^{\text{th}}$  root test – problems.

### Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

### TextBook

An Introduction to Real Analysis by Robert G.Bartle and Donald R. Sherbert, John Wiley and sons Pvt. Ltd

### Reference Books

- 1.Elements of Real Analysis by Shanthi Narayan and Dr. M.D. Raisinghania, S.Chand& Company Pvt. Ltd., New Delhi.
- 2.Principles of Mathematical Analysis by Walter Rudin, McGraw-Hill Ltd.

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**SECTION-A**

**Answer ALL FIVE Questions. Each question carries EIGHT marks.**

**5x8M=40M.**

1) a) State and prove Sandwich theorem or squeeze theorem.

(OR)

b) Apply Cauchy's general principle of Convergence to show that the sequence  $\{a_n\}$ , where

$$a_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} \text{ converges.}$$

2) a) State and prove Cauchy's  $n^{th}$  root test.

(OR)

b) Prove that  $\sum (-1)^{n-1} \frac{x^n}{n}$  is convergent for  $-1 < x \leq 1$ .

3) a) If  $f: I = [a, b] \rightarrow R$  is continuous on  $[a, b]$ , then prove that  $f$  is bounded on  $[a, b]$  and attains its Infimum & supremum.

(OR)

b) Examine for continuity the function  $f$  defined by  $f(x) = |x| + |x - 1|$  at  $x = 0, 1$ .

4) a) State and prove Lagrange's mean value theorem.

(OR)

b) Find 'C' of Cauchy's mean - value theorem for  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{\sqrt{x}}$  in  $[a, b]$  where  $0 < a < b$ .

5) a) Show that  $f(x) = 3x + 1$  is integrable on  $[1, 2]$  and  $\int_1^2 (3x + 1) dx = \frac{11}{2}$ .

(OR)

b) Prove that  $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$ .

**SECTION-B**

**Answer any FIVE of the following .Each question carries FOUR marks.**

**5x4M=20M**

6. Using Sandwich theorem ,Prove that  $\lim_{n \rightarrow \infty} \frac{\sin n}{n} = 0$ .

7. If the sequence  $\{S_n\}$  is convergent , then prove that  $\{S_n\}$  is a Cauchy Sequence

8. Test the convergence of  $\sum_{n=1}^{\infty} \frac{1}{3} \left( \frac{n+2}{n+3} \right)^n x^n \forall x > 0$ .

9. State and prove D'Alembert's Test.

10. Show that  $f: R \rightarrow R$  defined by  $f(x) = \frac{1}{1+|x|}$  is continuous and bounded. Find its supremum and infimum.

11. Discuss the continuity of  $f(x) = \frac{x e^{1/x}}{1+e^{1/x}}$  when  $x \neq 0$  and  $f(0) = 0$  at the origin. 12. Show that  $f(x) = x \sin \frac{1}{x}, x \neq 0; f(x) = 0, x = 0$  is continuous but not derivable at  $x = 0$ .

12. Examine the applicability of Rolle's theorem for  $f(x) = 1 - (x - 1)^{\frac{2}{3}}$  on  $[0, 2]$ .

13. Statement of Cauchy's Mean value Theorem.

14. Prove that  $f(x) = x^2$  is integrable on  $[0, a]$  and  $\int_0^a x^2 dx = \frac{a^3}{3}$ .

15. Show that  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2+r^2} = \frac{\pi}{4}$ .

**Course No-5 & Course Title- GROUP THEORY**

Course Objectives

1. To introduce students to the foundational concepts of algebraic structures with a focus on groups.
2. To develop an understanding of subgroups, cosets, and their relevance in group theory.
3. To explore the properties and significance of normal subgroups and their role in constructing quotient groups.
4. To study and apply the concepts of group homomorphisms, isomorphisms, and the fundamental theorem of homomorphism.
5. To examine the structure and properties of permutation and cyclic groups, including their role in group classification.

**Unit – 1 Groups**

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

**Unit – 2 Sub Groups**

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition-examples-criterion for a complex to be a subgroups; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – properties of Cosets – Index of a subgroups of a finite groups – Lagrange's Theorem.

**Unit – 3 Normal Subgroups**

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups Sub group of index 2 is a normal sub group

**Unit – 4 Homomorphisms**

Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

## **Unit – 5 Permutations and Cyclic Groups**

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

### **Adding:**

Unit-1- Order of an element of a group

Unit-2- Normalizer of an element of a group-Self-conjugate element of a group.

Unit-3-Simple group.

### **Activities:**

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

### **Text Book:**

Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, Krishna Prakashan Media Pvt. Ltd., Meerut.

### **Reference Books:**

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

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GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

II B.Sc. HONOURS, MATHEMATICS (MAJOR), SEMESTER-III

TIME: 3Hrs

COURSE-5, GROUP THEORY

Max.Marks:60

SECTION-A

Answer ALL FIVE Questions. Each question carries EIGHT marks.

5x8M=40M.

- 1) a) Show that the set  $Q^+$  of all positive rational numbers forms an abelian group under the

Composition defined by ' $\circ$ ' such that  $a \circ b = \frac{ab}{3}$  for  $a, b \in Q^+$ .

(OR)

- b) Prove that the set of  $n^{th}$  roots of unity under multiplication forms a finite abelian group.

- 2) a) If H and K are two subgroups of a group G then Prove that HK is a subgroup of G iff HK=KH.

(OR)

- b) State and prove Lagrange's Theorem.

- 3) a) Prove that a subgroup H of a group G is a Normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(OR)

- b) If H is a subgroup of G and N is normal subgroup of G, Then i) H intersection N is a normal subgroup ii) N is normal subgroup of HN.

- 4) a) State and prove fundamental theorem of Homomorphism of Groups.

(Or)

- b) If f is homomorphism of a group G then the Kernel of f is a normal subgroup of G.

- 5) a) State and prove Cayley's Theorem.

(OR)

- b) If P is a prime number then every group of order P is a Cyclic Group.

SECTION-B

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

- 6). Show that in a Group G, for  $a, b \in G$ ,  $(ab)^2 = a^2b^2$  iff G is abelian.

- 7). Prove that Cancellation laws hold in a group.

- 8). If  $H_1$  and  $H_2$  are two Subgroups of a Group G then  $H_1 \cap H_2$  is also a Subgroup of G.

- 9). Any two right cosets of a Subgroup are either disjoint or identical

- 10). Prove that a Subgroup H of a Group G is Normal , iff  $xHx^{-1} = H, \forall x \in G$ .

- 11). The intersection of any two normal subgroups of a group is a normal subgroup.

- 12). If f is a homomorphism from a group G into a group G' then  $\text{Ker } f$  is a normal subgroup of G.

- 13). The homomorphic image of a group is a group.

- 14). Examine whether the following Permutations is Even (or) Odd.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$ .

- 15). If G is an infinite cyclic group then G has exactly two generators.

Course No-6 & Course Title- NUMERICAL METHODS

Course Content

Unit – 1

The calculus of finite differences

The operators  $\Delta, \nabla, E$  - Fundamental theorem of difference calculus- properties of  $\Delta, \nabla, E$  and problems on them to express any value of the function in terms of the leading terms and the leading differences - relations between  $E$  and  $D$  - relation between  $D$  and  $\Delta$  - problems on one or more missing terms- Factorial notation- problems on separation of symbols- problems on Factorial notation.

Unit – 2: Interpolation with equal and unequal intervals

Derivations of Newton – Gregory Forward and backward interpolation and problems on them. Divided differences - Newton divided difference formula - Lagrange's and problems on them.

Unit – 3: Central Difference Interpolation formulae

Central Difference  $\delta, \mu, \sigma$  and relation between them - Gauss forward formula for equal Intervals - Gauss Backward formula - Stirlings formula - Bessel's formula –Everett's formula-and problems on the above formulae.

Unit – 4: Solution of Algebraic and Transcendental equation

Method for finding initial approximate value of the root - Bisection method - to find the solution of given equations by using (i) Regula Falsi method (ii) Iteration method (iii) Newton – Raphson's method and problems on them.

Unit – 5 : Curve Fitting

Least-squares curve fitting procedures - fitting a straight line-nonlinear curve fitting-curve fitting by assumption of exponentials

**GOVERNMENT COLLEGE FOR WOMEN (A),SRIKAKULAM**

**II B.Sc Mathematics**

**Semester-III**

**Course:-6 Numerical Methods**

**Time: 3 hours**

**Max Marks : 60**

**SECTION -A**

**Answer ALL FIVE Questions. Each question carries EIGHT Marks.**

**5X8=40M**

1. a)Find the Missing value in the following.

|      |     |    |     |       |
|------|-----|----|-----|-------|
| X    | 45  | 50 | 55  | 60    |
| F(x) | 3.0 | -  | 2.0 | 0.225 |

**(or)**

b).State and prove Fundamental theorem of Calculus.

2.a) Derive the Newton's forward formula

**(or)**

b).Estimate the sale for 1966 using the Newton's backward interpolation formula.

|                      |      |      |      |      |      |      |
|----------------------|------|------|------|------|------|------|
| Year x:              | 1931 | 1941 | 1951 | 1961 | 1971 | 1981 |
| Sale in thousands y: | 12   | 15   | 20   | 27   | 39   | 52   |

3.a)Derive the Gauss's backward formula for equal intervals.

**(or)**

b). Apply Gauss forward formula to find the value of  $u_9$  if  $u_0=14$  ;  $u_4=24$ ;  $u_8=32$  ;  $u_{16}=40$ .

4.a)Find the real roots of the equation  $x^3-9x+1=0$  by using Regular-Falsi Method.

**(or)**

b).Find a real roots of the equation  $x^3-2x-5=0$  by bisection method.

5.a)Fit a straight line of the form  $Y=a+bX$  to the following data.

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| X: | 1  | 2  | 3  | 4  | 5  | 6  |
| Y: | 55 | 46 | 40 | 38 | 33 | 30 |

**(or)**

b).Find the second degree parabola to the following data

|    |     |     |     |     |     |     |
|----|-----|-----|-----|-----|-----|-----|
| X: | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|----|-----|-----|-----|-----|-----|-----|

|    |    |     |     |     |     |     |
|----|----|-----|-----|-----|-----|-----|
| Y: | 72 | 110 | 158 | 214 | 290 | 380 |
|----|----|-----|-----|-----|-----|-----|

### SECTION-B

Answer any FIVE of the following . Each question Carries FOUR marks.

**5X4=20M**

6. Construct a forward difference table from the following data and evaluate  $\Delta^4 y_0$

|   |   |     |     |     |     |
|---|---|-----|-----|-----|-----|
| x | 0 | 1   | 2   | 3   | 4   |
| y | 1 | 1.5 | 2.2 | 3.1 | 4.6 |

7. Find the function whose first differences is  $x^3 + 4x^2 + 9x + 12$

8. Use Lagrange's interpolation formula to find  $y(0)$  to the data.

|   |    |    |    |    |
|---|----|----|----|----|
| X | 5  | 6  | 9  | 11 |
| Y | 12 | 13 | 14 | 16 |

9. Find the polynomial from given data

|   |   |   |    |     |
|---|---|---|----|-----|
| X | 0 | 1 | 2  | 5   |
| Y | 2 | 3 | 12 | 147 |

10. Prove that  $\Delta \equiv \frac{1}{2} \delta^2 + \delta \left( 1 + \frac{\delta^2}{4} \right)^{\frac{1}{2}}$

11. State and prove Stirling's formula.

12. Find a root of the equation  $x^3 - x^2 - 1 = 0$  by using iteration method.

13. Find the real root of the equation  $4x^3 + 9x^2 + 9x + 5 = 0$  using bisection method.

14. Fit a curve  $y = ae^{bx}$  to the given data

|   |     |    |     |
|---|-----|----|-----|
| X | 0   | 2  | 4   |
| Y | 5.1 | 10 | 3.4 |

15. Fit a Straight line of the following data

|   |   |   |   |    |
|---|---|---|---|----|
| X | 0 | 1 | 2 | 3  |
| y | 2 | 5 | 8 | 11 |

**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-III**

**Credits-4**

**Hrs/Wk-5**

**Course No-7 & Course Title- LAPLACE TRANSFORMS**

**Unit – 1 LAPLACE TRANSFORMS – I**

Definition of Laplace Transform - Linearity Property - Piecewise Continuous Function - Existence of Laplace Transform - Functions of Exponential order and of Class A.

**Unit – 2 LAPLACE TRANSFORMS – II**

First Shifting Theorem, Second Shifting Theorem, Change of Scale Property, Laplace transform of the derivative of  $f(t)$ , Initial value theorem and Final value theorem.

**Unit – 3 LAPLACE TRANSFORM – III**

Laplace Transform of Integrals - Multiplication by  $t$ , Multiplication by  $t^n$  - division by  $t$  -Laplace transform of Bessel Function - Laplace Transform of Error Function – Laplace transform of Sine and Cosine integrals- **Laplace transform of periodic function.**

**Unit – 4 INVERSE LAPLACE TRANSFORMS – I**

Definition of Inverse Laplace Transform - Linearity Property - First Shifting Theorem -Second Shifting Theorem - Change of Scale property - use of partial fractions - Examples.

**Unit – 5 INVERSE LAPLACE TRANSFORMS – II**

Inverse Laplace transforms of Derivatives - Inverse Laplace Transforms of Integrals -Multiplication by Powers of ' $p$ ' - Division by powers of ' $p$ ' - Convolution Definition -Convolution Theorem - proof and Applications - Heaviside's Expansion theorem and its Applications.

**Activities**

Seminar/ Quiz/ Assignments/ Applications of Laplace Transforms to Real life Problem  
/ProblemSolving Sessions.

**GOVERNMENT COLLEGE FOR FOR WOMEN(A),SRIKAKULAM**

**II B.Sc. HONOURS, MATHEMATICS(MAJOR), SEMESTER-III**

**TIME: 3Hrs**

**COURSE 7 -Laplace transforms**

**Max.Marks:60**

**SECTION-A**

**Answer All following questions .Each question carries eight marks.**

**5X8M=40M**

1) a) Find the laplace transformation of the following function

$$e^{3t} - 2e^{-2t} + \sin 2t + \cos 3t + \sinh 3t - 2\cosh 4t + 9$$

OR

b.) Using expansion  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ , show that  $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$

2.) a) Find the laplace transform of the function i)  $te^{-t \sin 2t}$  ii)  $L\{t^2 \cos 3t\}$

OR

b) Find the  $L\{F(t)\}$ , where  $F(t)$  is a periodic function of period  $2\pi$  and it is given by

$$F(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

3.) a) Find the inverse laplace transform of  $\frac{4p+5}{(p-1)^2(p+2)}$

OR

b) State and prove convolution theorem

4.) a) Solve  $y'' - ty' + y = 1$  if  $y(0)=1, y'(0)=2$

OR

b.) Find the inverse laplace transform of  $\frac{4}{(p+1)(p+2)}$

5.) a) state and prove change scale property in inverse laplace transforms

OR

b) state and prove the Heaviside's expansion theorem.

### **SECTION-B**

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

6.Find the  $L\{7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2\}$

7.Find the Laplace transform of  $(\sin t + \cos t)^2$

8.Find  $L\{(t^2 - 3t + 2)\sin 3t\}$

9.find  $L\{t \sin at\}$

10.evaluate  $L\{\sin at - t \cos at\}$

11.Solve  $(D^2 - 2D + 2)y = 0, y = Dy = 1, \text{ when } t = 0$

12. find the Laplace transform of  $\frac{\sin at}{t}$

13.State and prove initial value theorem.

14. .Find the inverse Laplace transform of  $\frac{3p-8}{4p^2+25}$

15.Find the inverse Laplace transform of  $\frac{3p-2}{p^2-4p+20}$

**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**SEMESTER-III**

**Credits-4**

**Hrs/Wk-5**

**Course No-8 & Course Title- Special Functions**

**UNIT 1- Beta and Gamma functions, Chebyshev polynomials**

1. Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.
2. Another form of Beta Function, Relation between Beta and Gamma Functions.
3. Chebyshev polynomials, orthogonal properties of Chebyshev polynomials of first kind, recurrence relations, generating functions for Chebyshev polynomials.

**UNIT-2 Power series and Power series solutions of ordinary differential equations**

1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series
2. Introduction of power series solutions of ordinary differential equation
3. Ordinary and singular points, regular and irregular singular points, power series solution.

**Unit – 3 Hermite polynomials**

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.
2. Other forms for Hermite Polynomials, Rodrigue's formula for Hermite Polynomials, to find first few Hermite Polynomials.
3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

**Unit – 4 Legendre polynomials**

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree  $n$ , generating function of Legendre polynomials.
2. Definition of  $P_n(x)$  and  $Q_n(x)$ , General solution of Legendre's Equation (derivations not required) to show that  $P_n(x)$  is the coefficient of  $h^n$ , in the expansion of  $(1-2xh+h^2)^{-1/2}$
3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

### **Unit – 5 Bessel's equation**

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order  $n$ , Bessel's function of the second kind of order  $n$ .
2. Integration of Bessel's equation in series form, Definition of  $J_n(x)$ , recurrence formulae for  $J_n(x)$ .
3. Generating function for  $J_n(x)$ , orthogonality of Bessel functions.

GOVERNMENT COLLEGE FOR FOR WOMEN(A),SRIKAKULAM

II B.Sc. HONOURS, MATHEMATICS(MAJOR), SEMESTER-III

TIME: 3Hrs

COURSE 8 - Special Functions

Max.Marks:60

**SECTION-A**

Answer All following questions .Each question carries eight marks.

5X8M=40M

1 a)Show that  $\beta(l, m) = \frac{\Gamma l \Gamma m}{\Gamma l+m}$ .

(OR)

b)Prove that  $\Gamma n \Gamma(1 - n) = \frac{\pi}{\sin n\pi}$ .

2 a)Find the series solution about the origin of the D.E  $x^2 y'' + 6xy' + (6 + x^2) = 0$

(OR)

b)Solvey" - xy' + x^2y = 0 in power of x.

3 a)State and prove orthogonal property of  $H_n(x)$ .

(OR)

b)Prove that  $H_n(x) = 2^n \{exp\left(-\frac{1}{4} \frac{d^2}{dx^2}\right)\} x^n$ .

4 a)State and prove Generating Function of legendre polynomial.

(OR)

b)Prove that  $\int_{-1}^1 x^n p_n(x) dx = \frac{2^{n+1}(n!)^2}{(2n+1)!}$

5 a)Prove that  $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n$

(OR)

b)Prove that i) $J_{-n}(x) = (-1)^n J_n(x)$  and ii) $J_n(-x) = (-1)^n J_n(x)$ .

**SECTION - B**

Answer **any FIVE** questions. Each question carries **FOUR**marks

5 X 4 =20M

6)Compute  $\Gamma\left(-\frac{3}{2}\right)$  and  $\Gamma\left(\frac{5}{2}\right)$ .

7)Evaluate  $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx$

8)Prove that  $2^n \Gamma\left(n + \frac{1}{2}\right) = 1.3.5 \dots (2n - 1) \Gamma \pi$ .

9)Solve by power series method  $y' - y = 0$ .

10)Prove that i) $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$  and ii) $H_{2n+1}(0) = 0$ .

11)Evaluate  $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x) dx$ .

12)Show that  $P_n(1) = 1$  and  $P_n(-x) = (-1)^n P_n(x)$ .

13)Show that  $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$ .

14)Show that  $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$ .

15)Write the series from expression for  $J_0(x)$ .

# GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM

**SEMESTER-IV**

**Credits-4**

**Hrs/Wk-5**

## **Course No-9 & Course Title- RING THEORY**

### Course Objectives

The course aims to:

1. Introduce the fundamental concepts and properties of rings, fields, and integral domains.
2. Explain the structure and significance of subrings and ideals, including prime and maximal ideals.
3. Construct quotient rings and develop composition tables for finite rings.
4. Explore ring homomorphisms, isomorphisms, and apply the fundamental theorems of ring homomorphisms.
5. Study polynomial rings, including operations, division algorithm, irreducibility, and ideal structures.

### **Course Content**

#### **Unit 1-Rings and Fields**

Definition of a ring and Examples –Basic properties – Boolean rings - Fields – Divisors of 0 and Cancellation Laws– Integral Domains – Division ring - The Characteristic of a Ring, Integral domain and Field – Non Commutative Rings - Matrices over a field .

#### **Unit – 2 Subrings and Ideals**

Definition and examples of Subrings – Necessary and sufficient conditions for a subset to be a Subring— Algebra of Subrings – Centre of a ring – left, right and two sided ideals – Algebra of ideals –Equivalence of a field and a commutative ring without proper ideals

#### **Unit 3: Principal ideals and Quotient rings**

Definition of a Principal ideal ring(Domain) – Every field is a PID – The ring of integers is a PID – Example of a ring which is not a PIR – Cosets – Algebra of cosets – Quotient rings – Construction of composition tables for finite quotient rings of the ring  $\mathbb{Z}$  of integers and the ring  $\mathbb{Z}_n$  of integers modulo  $n$ .

#### **Unit 4:Homomorphism of Rings**

Homomorphism of Rings – Definition and Elementary properties – Kernel of a homomorphism – Isomorphism – Fundamental theorems of homomorphism of rings – Maximal and prime Ideals –

PrimeFields

### **Unit – 5: Rings of Polynomials**

Polynomials in an indeterminate – The Evaluation morphism -- The Division Algorithm in  $F[x]$  – Irreducible Polynomials – Ideal Structure in  $F[x]$  – Uniqueness of Factorization  $F[x]$ .

#### Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

#### Textbook

ModernAlgebra by A.R. Vasishta and A.K. Vasishta, KrishnaPrakashan Media Pvt. Ltd.

#### Referencebooks

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others, Pearson Education India

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**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM**

**II B.Sc. HONOURS MATHEMATICS (MAJOR), SEMESTE**

**TIME: 3Hrs**

**COURSE-9, RING THEORY**

**Max.Marks:60**

**SECTION-A**

**Answer ALL FIVE Questions. Each question carries EIGHT marks.**

**5x8M=40M.**

1) a) Every finite integral domain is a field

(OR)

b) The characteristic of an integral domain is either a prime or zero

2) a) Let  $S$  be a non empty subset of a ring  $R$  then  $S$  is a subring of  $R$  iff  $a-b \in S$  and  $ab \in S$  for all  $a, b \in S$

(OR)

b) If  $S_1$  and  $S_2$  are two Subrings of a Ring  $R$  then  $S_1 \cap S_2$  is also a Subring of  $R$ .

3) a) Prove that Every Euclidean ring is a principal ideal ring.

(OR)

b) Prove that Every ideal of  $\mathbb{Z}$  is a principal ideal

4) a) State and prove fundamental theorem of Homomorphism of Rings.

(Or)

b) If  $f$  is homomorphism of a ring  $R$  then the Kernel of  $R$  is an ideal of  $R$ .

5) a) State and prove the Division Algorithm in ring of polynomials.

(OR)

b) An element  $a \in F$  where  $F$  is a field, is a zero of  $f(x) \in F[x]$  iff  $(x-a)$  is a factor of  $f(x)$  in  $F[x]$ .

**SECTION-B**

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

6. Every field is an integral domain

7. prove that every Boolean ring is abelian

8. The intersection of two subrings of a ring  $R$  is a subrings.

9. A field has no proper non trivial ideals.

10. A field is a principal ideal ring

11. If  $U$  is an ideal of the ring  $R$  and  $a, b \in R$  then prove that  $a+U=b+U \Rightarrow a-b \in U$

12. If  $f$  is a homomorphism from a ring  $R$  into a ring  $R'$  then  $\text{Ker } f$  is an ideal of  $R$ .

13. The homomorphic image of a ring is a ring.

14. If  $f(x), g(x)$  are two non-zero polynomials of  $F[x]$  where  $F$  is a field then  $\deg(f(x).g(x)) = \deg f(x) + \deg g(x)$ .

15. Find the units in  $\mathbb{Z}_7[x]$ .

# GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM

SEMESTER-IV

Credits-4

Hrs/Wk-5

## Course No-10 & Course Title- INTRODUCTION TO REAL ANALYSIS

### Course Objectives

This course is designed to:

1. Develop deep understanding of infinite series with non-negative terms and apply various convergence tests.
2. Introduce the concepts of limits and continuity, including their behavior at finite and infinite points.
3. Explore types of discontinuities and apply fundamental theorems related to continuous functions.
4. Understand differentiability and apply Mean Value Theorems in problem-solving.
5. Introduce Riemann integration and explore key properties and theorems of integrable functions.

### **Unit – 1 REALNUMBERS,REAL SEQUENCES**

The algebraic and order properties of  $\mathbb{R}$  - Absolute value and Real line - Completeness property of  $\mathbb{R}$  - Applications of supremum property - intervals. **(No question is to be set from this portion)** Sequences and their limits -Range and Boundedness of Sequences - Limit of a sequence and Convergent sequence -The Cauchy's criterion - properly divergent sequences - Monotone sequences - Necessary and Sufficient condition for Convergence of Monotone Sequence - Limit Point of Sequence –Sub sequences and the Bolzano-weierstrass theorem – Cauchy Sequences – Cauchy's general principle of convergence.

### **Unit – 2 INFINITE SERIES**

Introduction to series –convergence of series -Cauchy's general principle of convergence for series tests for convergence of series - Series of non-negative terms - P-test - Cauchy's  $n^{\text{th}}$  root test -D'Alembert's Test-Alternating Series–Leibnitz Test.

### **Unit –3 LIMIT & CONTINUITY**

Real valued Functions - Boundedness of a function - Limits of functions - Some extensions of the limit concept - Infinite Limits - Limits at infinity **(No question is to be set from this portion)**.Continuous functions - Combinations of continuous functions - Continuous Functions on intervals - uniform

continuity.

#### **Unit – 4 DIFFERENTIATION AND MEAN VALUE THEORMS**

The derivability of a function at a point and on an interval - Derivability and continuity of a function  
–Mean value Theorems - Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem.

#### **Unit – 5 RIEMANN INTEGRATION**

Riemann Integral - Riemann integral functions - Darboux theorem -Necessary and sufficient condition for R integrability - Properties of integrable functions - Fundamental theorem of integral calculus - integral as the limit of a sum - Mean value Theorems.

#### Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

#### TextBook

An Introduction to Real Analysis by Robert G. Bartle and Donald R. Sherbert, John Wiley and sons Pvt. Ltd

#### Reference Books

1. Elements of Real Analysis by Shanthi Narayan and Dr. M.D. Raisinghania, S.Chand & Company Pvt. Ltd., New Delhi.
2. Principles of Mathematical Analysis by Walter Rudin, McGraw-Hill Ltd.

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**GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS), SRIKAKULAM**

**II B.Sc. HONOURS, MATHEMATICS (MAJOR), SEMESTER-IV**

**TIME:3Hrs    COURSE-10, INTRODUCTION TO REAL ANALYSIS    MAX MARKS : 60**

**SECTION-A**

**Answer ALL FIVE Questions. Each question carries EIGHT marks.**

**5x8M=40M.**

1) a) State and prove Sandwich theorem or squeeze theorem.

(OR)

b) Apply Cauchy's general principle of Convergence to show that the sequence  $\{a_n\}$ , where

$$a_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} \text{ converges.}$$

2) a) State and prove Cauchy's  $n^{th}$  root test.

(OR)

b) Prove that  $\sum (-1)^{n-1} \frac{x^n}{n}$  is convergent for  $-1 < x \leq 1$ .

3) a) If  $f: I = [a, b] \rightarrow R$  is continuous on  $[a, b]$ , then prove that  $f$  is bounded on  $[a, b]$  and attains its Infimum & supremum.

(OR)

b) Examine for continuity the function  $f$  defined by  $f(x) = |x| + |x - 1|$  at  $x = 0, 1$ .

4) a) State and prove Lagrange's mean value theorem.

(OR)

b) Find 'C' of Cauchy's mean - value theorem for  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{\sqrt{x}}$  in  $[a, b]$  where  $0 < a < b$ .

5) a) Show that  $f(x) = 3x + 1$  is integrable on  $[1, 2]$  and  $\int_1^2 (3x + 1) dx = \frac{11}{2}$ .

(OR)

b) Prove that  $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$ .

## SECTION-B

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20M

6. Using Sandwich theorem ,Prove that  $\lim_{n \rightarrow \infty} \frac{\sin}{n} = 0$ .

7. If the sequence  $\{S_n\}$  is convergent , then prove that  $\{S_n\}$  is a Cauchy Sequence

8. Test the convergence of  $\sum_{n=1}^{\infty} \frac{1}{3} \left( \frac{n+2}{n+3} \right)^n x^n \forall x > 0$ .

9. State and prove D'Alembert's Test.

10. Show that  $f: R \rightarrow R$  defined by  $f(x) = \frac{1}{1+|x|}$  is continuous and bounded. Find its supremum and infimum.

11. Discuss the continuity of  $f(x) = \frac{xe^{1/x}}{1+e^{1/x}}$  when  $x \neq 0$  and  $f(0) = 0$  at the origin. 12. Show that  $f(x) = x \sin \frac{1}{x}, x \neq 0; f(x) = 0, x = 0$  is continuous but not derivable at  $x = 0$ .

12 .Examine the applicability of Rolle's theorem for  $f(x) = 1 - (x - 1)^{\frac{2}{3}}$  on  $[0,2]$ .

13. Statement of Cauchy's Mean value Theorem.

14. Prove that  $f(x) = x^2$  is integrable on  $[0, a]$  and  $\int_0^a x^2 dx = \frac{a^3}{3}$ .

15. Show that  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2+r^2} = \frac{\pi}{4}$ .

**GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM**

**SEMESTER-IV**

**Credits-4**

**Hrs/Wk-5**

**Course No-11 & Course Title- INTEGRAL TRANSFORMS WITH APPLICATIONS**

**Course Content**

**Unit – 1 Application of Laplace Transform to solutions of Differential Equations**

Solutions of ordinary Differential Equations - Solutions of Differential Equations with constants coefficients - Solutions of Differential Equations with Variable coefficients.

**Unit – 2 Application of Laplace Transform to solutions of Differential Equations**

Solutions of Simultaneous Ordinary Differential equations - Solutions of Partial Differential Equations.

**Unit – 3 Application of Laplace Transforms to Integral Equations**

Definitions of Integral Equations - Abel's Integral Equation - Integral Equation of Convolution Type - Integral Differential Equations - Application of L.T. to Integral Equations.

**Unit – 4 Fourier Transforms - I**

Definition of Fourier Transform - Fourier sine Transform - Fourier cosine Transform - Linear Property of Fourier Transform - Change of Scale Property for Fourier Transform - sine Transform and cosine transform shifting property - Modulation theorem.

**Unit – 5 Fourier Transforms – II**

Definition of Convolution - Convolution theorem for Fourier transform - Parseval's Identity - Relationship between Fourier and Laplace transforms - problems related to Integral Equations -Finite Fourier Transforms - Finite Fourier Sine Transform - Finite Fourier Cosine Transform - Inversion formula for sine and cosine transforms only - statement and related problems

**GOVERNMENT COLLEGE FOR WOMEN,(AUTONOMOUS,)SRIKAKULAM**  
**II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV**  
**COURSE-11, INTEGRAL TRANSFORMS WITH APPLICATIONS**

**TIME: 3Hrs**

**Max.Marks:60**

**SECTION – A**

**Answer all five questions each question carries ten marks.**

**5x8M=40M**

- 1) a) By using Laplace transform ,solve  
 $(D^2-D-2)y = 20 \sin 2t$  ,  $y=-1, Dy=2$  when  $t=0$ .  
**or**  
 b ) Solve  $ty''+y'+4ty=0$  if  $y(0) = 3$  ,  $y'(0)=0$ .
- 2.) a) by using Laplace transforms solve  
 $(D^2-3)x-4y=0$  ,  $x+(D^2+1)y=0$  ,  $t>0$  if  $x=y=Dy=0$  ,  $Dx=2$ .  
**or**  
 b) solve  $\partial^2 y / \partial x^2 - \partial^2 y / \partial t^2 = xt$  where  $y=0 = \partial y / \partial t$  at  $t=0$  and  $y(0,t)=0$ .
- 3) a) solve the integral equation  
 $F(t)=a \sin t - 2 \int F(u) \cos(t-u) du$   
**or**  
 b.) solve the integral equation  $\int F(u) du / (t-u)^{1/3} = t(1+t)$ .
- 4.) a) show that the fourier transform of  $f(x)=e^{-x^2/2}$  is  $e^{-p^2/2}$ .  
**or**  
 b) find the fourier transform of  $f(x)=1-x^2$  if  $|x| < 1$  ,  $f(x)=0$  if  $|x| > 1$ .  
 And hence evaluate  $\int (x \cos x - \sin x / x^3) \cos x / 2 dx$ .
- 5) a) if  $f(x)= 1$  if  $|x| < a$  ,  $f(x)=0$  if  $|x| > a$  and  $f(p)=2 \sin pa / p$  ( $p \neq 0$ )  
 And prove that  $\int \sin^2 ax / x^2 dx = \pi a / 2$ .  
**or**  
 b) show that  $\int \cos \lambda x / \lambda^2 + 1 d\lambda = \pi / 2 e^{-x}$  ,  $x > 0$ .

**SECTION-B**

**ANSWER ANY FIVE OF THE FOLLOWING.**

**5x4M=20M**

- 6) Solve  $dy/dx+y=1$  given that  $y=2$  when  $t=0$ .
- 7) .Solve  $(D^2-3D+2)y=1-e^{2x}$  if  $y=1, Dy=0$  when  $t=0$ .
- 8) If  $y(x,t)$  is a function of  $x$  and  $t$  , prove that  
 I)  $L\{\partial y / \partial t\} = P \tilde{Y}(x,p) - y(x,0)$  II)  $L\{\partial^2 y / \partial t^2\} = P^2 \tilde{Y}(x,p) - p y(x,0) - y_t(x,0)$ .
- 9) Solve the integral equation  $F(t)= e^{-t} - 2 \int \cos(t-u) du$ .
- 10) Find the fourier sine transform of  $f(x)$  if  $f(x)=0$  if  $0 < x < a$  ,  $f(x)=x$  if  $a < x < b$  ,  $f(x)= 0$  if  $x > b$ .
- 11) Find the fourier transform  $f(x)$  if  $F(x)=\sqrt{2}\pi/2a$  if  $|x| < a$  ,  $f(x)=0$  if  $|x| > 0$ .
- 12).Find the fourier sine and cosine transform of  $f(x)=x$ .
- 13).Solve the integral equation  $\int f(x) \cos \lambda x dx = e^{-\lambda}$ .
- 14) Solve  $y''(t)+y(t)=t$  given that  $y'(0)=1$  ,  $y(0)=0$ .
- 15) .Find the finite fourier sine and cosine transform of  $f(x)=1$ .

**GOVERNMENT COLLEGE FOR WOMEN,(A),SRIKAKULAM**

**Multidisciplinary Course w.e.f. AY 2023-24**

**SEMESTER-IV**

**BASIC STATISTICS**

Credits: 2

2 hrs/week

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**Unit I:** Meaning, scope and limitations of Statistics

*Collection of data:* Primary and Secondary, Classification and Tabulation, Construction of frequency distribution.

*Graphical Representation:* Histogram, Bar, Pie and Frequency polygon.

**Unit II:** *Measures of Central Tendency:* Features of good average, Arithmetic Mean, Median, Mode. Empirical relationship between Mean Median and Mode and skewness based on central values. (8hrs)

**Unit III:** *Measures of Dispersion:* Range, Quartile Deviation(QD), Mean Deviation(MD), Variance, Standard Deviation(SD), relationship between QD, MD and SD. Familiarization of the concepts relating to Correlation and Linear Regression line. (8hrs)

**Books for Study:**

1. Statistics (Theory, Methods, Application) D C Sancheti, V K Kapoor, Sultan Chand and Sons, New Delhi
2. Statistical Methods, S.P. Gupta, Sultan Chand and Sons, New Delhi
3. Statistics (Theory and Practice) B.N Gupta, Sahitya Bhavan, Agra

Web sites for free download books for Statistics

<https://www.pdfdrive.com/introduction-to-statistics-books.html>

<http://www.freebookcentre.net/SpecialCat/Free-Statistics-Books-Download.html> <https://bookboon.com/en/statistics-ebooks>

[http://onlinestatbook.com/Online\\_Statistics\\_Education.pdf](http://onlinestatbook.com/Online_Statistics_Education.pdf)

**GOVERNMENT COLLEGE FOR WOMEN,(A),SRIKAKULAM**  
**BLUE PRINT FOR QUESTION PAPER PATTERN**  
**MULTIDISCIPLINARY COURSE**  
**BASIC STATISTICS**

| <b>Units</b> | <b>Course Content</b>                       | <b>E.Q)(10M)</b> | <b>S.A.Q)(5M)</b> | <b>Total</b> |
|--------------|---------------------------------------------|------------------|-------------------|--------------|
| <b>I</b>     | Meaning scope and limitations of statistics | 1                | 2                 | 20           |
| <b>II</b>    | Measures of central tendency                | 2                | 3                 | 35           |
| <b>III</b>   | Measure of dispersion                       | 2                | 3                 | 35           |
|              | <b>TOTAL</b>                                | <b>5</b>         | <b>8</b>          | <b>90</b>    |

**S.A.Q.** = Short answer questions (5 marks)

**E.Q.** = Essay questions (10 marks)

Short answer questions :4X5M =20M

Essay questions :3X10M =30M

Total Marks =50M

**Note:**

1. The question paper setter are requested to kindly adhere to the format given in the above table.
2. The question paper setter are also requested to set the questions in the following way:
  - a. 70 % of questions – Remembering and Understanding type questions
  - b. 30 % of questions – Applying, Evaluating, Analyzing and Creating type questions

**GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM**

**Department Of MATHEMATICS**

**B.Sc/B.A/B.COM (w.e.f. AY 2023-24)**  
**BASIC STATISTICS----Semesters –IV**

**TIME: 2Hrs.**

**MARKS:50**

|                                                                           |  |                        |
|---------------------------------------------------------------------------|--|------------------------|
| <b>SECTION-A</b><br><b>(Essay Questions)</b>                              |  |                        |
| <b>Instructions to the paper setter: Internal choice from each unit</b>   |  |                        |
| <b>Answer THREE of the following question :</b>                           |  | <b>3x10M=30 Marks</b>  |
| <b>1.</b>                                                                 |  | <b>(from Unit-I)</b>   |
| <b>2.</b>                                                                 |  | <b>(from Unit-II)</b>  |
| <b>3.</b>                                                                 |  | <b>(from Unit-III)</b> |
| <b>4.</b>                                                                 |  |                        |
| <b>5.</b>                                                                 |  |                        |
| <b>SECTION –B</b><br><b>(Short Answer Questions)</b>                      |  |                        |
| <b>Instructions to the paper setter: Set TWO questions from each unit</b> |  |                        |
| <b>Answer any FOUR of the following questions</b>                         |  | <b>4x5M=20Marks</b>    |
| <b>6. (from Unit-I)</b>                                                   |  |                        |
| <b>7. (from Unit-I)</b>                                                   |  |                        |
| <b>8. (from Unit-II)</b>                                                  |  |                        |
| <b>9. (from Unit-II)</b>                                                  |  |                        |
| <b>10. (from Unit-II)</b>                                                 |  |                        |
| <b>11. (from Unit-III)</b>                                                |  |                        |
| <b>12. (from Unit-III)</b>                                                |  |                        |
| <b>13. (from Unit-III)</b>                                                |  |                        |

**Signed and approved by:**

**Chair Person**

**Subject Expert**  
(Univ. Nominee)

**Subject Expert**  
(Nominated by Academic Council)

**Subject Expert**

**Member of Industrialist**

**Member of Alumni**

**FACULTY MEMBERS**

(Nominated by Academic Council)

# GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

**SEMESTER-V**

**Credits-4**

**Hrs/Wk-5**

## **Course No-12 & Course Title- LINEARALGEBRA**

### Course Objectives

1. To introduce the fundamental concepts of vector spaces, sub-spaces, and their algebraic structure.
2. To develop an understanding of basis and dimension of vector spaces and their associated theorems.
3. To explore linear transformations and their properties, including rank, nullity, and the Rank-Nullity Theorem.
4. To apply the Cayley-Hamilton Theorem to compute powers and inverses of matrices without using direct methods.
5. To understand the structure of inner product spaces and study orthogonality and related geometric properties.

### **Course Content**

#### **UNIT– I-Vector Spaces-I**

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - addition and scalar multiplication of Vectors - internal and external composition - Null space - Vector subspaces -Algebra of subspaces - Linear Sum of two subspaces - linear combination of Vectors- Linear span Linear independence and Linear dependence of Vectors.

#### **UNIT–II-Vector Spaces-II**

Basis of Vector space-Finite dimensional Vector spaces-basis extension-co-ordinates-Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space.

#### **UNIT–III-Linear Transformations**

Linear transformations-linear operators-Properties of L.T-sum and product of L.Ts-Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformations - Rank- Nullity Theorem.

#### **UNIT–IV-Matrices**

Characteristic equation-Characteristic Values-Characteristic vectors of a square matrix-Cayley Hamilton Theorem – problems on Cayley-Hamilton Theorem.

#### **UNIT–V-Inner product space**

Inner product spaces- Euclidean and unitary spaces- Norm or length of a Vector- Schwartz inequality- Triangle Inequality- Parallelogram law- Orthogonality- Orthonormal set- Problems on Gram– Schmidt

orthogonalisation process - Bessel's inequality.

### Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

### TextBook

LinearAlgebrabyJ.N.SharmaandA.R.Vasishta,published by Krishna Prakashan Media (P) Ltd.

### ReferenceBooks

1. LinearAlgebrabyStephenH.Friedberget.al.publishedbyPrenticeHallofIndia Pvt. Ltd. 4<sup>th</sup> Edition, 2007
2. Linear Algebra byKenneth Hoffman and RayKunze, published by Pearson education low priced edition), New Delhi.
3. Matrices by Shanti Narayana, published byS. Chand Publications

**GOVERNMENT COLLEGE FOR WOMEN,(AUTONOMOUS),SRIKAKULAM**

**III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V**

**TIME: 3Hrs**

**COURSE-12, LINEARALGEBRA**

**Max.Marks:60**

**SECTION – A**

**Answer all five questions each question carries ten marks.**

**5x8M=40M**

1 (a) Let  $V(F)$  be a vector space .let  $W$  be a non-empty subset of  $V$ . The necessary and sufficient condition for  $W$  to be a subspace of  $V$  is  $a, b \in F$  and  $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$ .

(OR)

(b) If  $W_1$  and  $W_2$  are two subspaces of a vector space  $V(F)$  then  $L(W_1 \cup W_2)=W_1+W_2$ .

2. (a) Let  $W_1$  and  $W_2$  be two subspaces of a finite dimensional vector space. Then

$$\dim (W_1+W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2).$$

(OR)

(b) Find a basis for  $\mathbb{R}^4$  write  $(3,2,5,4)(6,3,5,2)$  as numbers.

3. (a) Find  $T(x,y,z)$  where  $T:\mathbb{R}^3 \rightarrow \mathbb{R}$  is defined by  $T(1,1,1)=3$  ;  $T(0,1,-2)=1$  ;  $T(0,0,1)=-2$ .

(OR)

(b) If  $T:V_4(\mathbb{R}) \rightarrow V_3(\mathbb{R})$  linear transformation defined by  $T(a,b,c,d)=(a-b+c+d, a+2c-d, a+b+3c-3d)$  for  $a,b,c,d \in \mathbb{R}$  then verify  $\rho(T) + \vartheta(T) = \dim V_4(\mathbb{R})$ .

4. (a) Find the characteristic roots and corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

5. (a) State and prove Cauchy-Schwarz'S inequality.

(OR)

(b)State and prove Bessel's inequality.

## SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

- 6) Prove that the intersection of any two subspaces  $W_1$  and  $W_2$  of vector space  $V(F)$  is also a subspace.
- 7) If  $\alpha, \beta, \gamma$  are linearly independent vectors of  $V(R)$ . Show that  $\alpha + \beta, \beta + \gamma, \gamma + \alpha$  are also linearly independent.
- 8) If  $\alpha=(1,-1,0)$   $\beta=(2,1,3)$  find a basis for  $R^3$  containing  $\alpha$  and  $\beta$ .
- 9) Let  $W_1$  and  $W_2$  be two subspaces of  $R^4$  given by  $W_1=\{(a,b,c,d):b-2c+d=0\}$ ,  $W_2=\{(a,b,c,d):a=d,b=2c\}$  find the basis and dimension of  $W_1$ .
- 10) The mapping  $T: V_3(R) \rightarrow V_2(R)$  is defined by  $T(x, y, z)=(x-y, x-z)$  show that  $T$  is a linear transformation.
- 11) Let  $U(F)$  and  $V(F)$  be two vector spaces. Let  $T:U(F) \rightarrow V(F)$  be a linear transformation then the range set  $R(T)$  is a subspace of  $V(F)$ .
- 12) Prove that the square matrices  $A$  and  $A^T$  have the same characteristic values.
- 13) Find the characteristic roots of the matrix  $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 4 & -1 \\ 2 & -1 & 4 \end{bmatrix}$ .
- 14) State and prove Triangle inequality.
- 15) State and prove Parallelogram law.

# GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

## Course No-13 & Course Title-VECTOR CALCULUS

### Course Objectives

1. To introduce the concept of vector differentiation and the use of vector operators such as gradient, divergence, and curl.
2. To extend the concept of definite integrals to functions of multiple variables through double and triple integrals.
3. To apply double and triple integrals in evaluating areas and volumes in different coordinate systems.
4. To provide knowledge on evaluating line, surface, and volume integrals using vector calculus techniques.
5. To understand and apply major vector integration theorems like Gauss's Divergence Theorem, Green's Theorem, and Stokes's Theorem.

### Course Content

#### Unit-1

Introduction –Double integrals –Evaluation of double integrals –Properties of double integrals –  
Region of integration – double integration in Polar Co-ordinates –  
Change of variables in double integrals –change of order of integration.

#### **Unit-2 Multiple integrals-II**

Triple integral –region of integration –change of variables –Plane areas by double integrals –  
surface area by double integral –Volume as a double integral, volume as a triple integral.

#### **Unit-3 Vector differentiation**

Vector differentiation–ordinary-derivatives of vectors – Differentiability–Gradient–Divergence- Curl  
operators – Formulae involving these operators .

### **Unit–4 Vector integration**

Line Integrals with examples – Surface Integral with examples - Volume integral with examples.

### **Unit–5 Vector integration applications**

Gauss theorem and applications of Gauss theorem - Green's theorem in plane and applications of Green's theorem - Stokes's theorem and applications of Stokes theorem.

#### Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

#### Text Book

A text Book of Higher Engineering Mathematics by B.S.Grawal, Khanna Publishers, 43<sup>rd</sup> Edition

#### Reference Books

1. Vector Calculus by P.C. Matthews, Springer Verlag publications.
2. Vector Analysis by Murray Spiegel, Schaum Publishing Company, NewYork

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**GOVERNMENT COLLEGE FOR WOMEN,(AUTONOMOUS,)SRIKAKULAM**

**III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V**

**TIME: 3Hrs**

**COURSE-13, VECTOR CALCULUS**

**Max.Marks:60**

**SECTION – A**

**Answer all five questions each question carries ten marks.**

**5x8M=40M**

1. a) Evaluate  $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$  by changing the order of integration.

OR

b) By changing into polar co-ordinates evaluate  $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$  over the angular region between the circles  $x^2 + y^2 = a^2, x^2 + y^2 = b^2$  ( $b > a$ ).

2. a) Evaluate the integral  $\iiint xy dx dy dz$  where R is the positive octant of the sphere  $x^2 + y^2 + z^2 = a^2$ .

OR

b) If R is region in the first octant bounded by the sphere  $x^2 + y^2 + z^2 = a^2$ , evaluate the integral  $\iiint x + y + z dx dy dz$  by changing to spherical polar coordinates.

3. a) Prove that  $\text{curl} (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \text{div} \mathbf{B} - \mathbf{B} \text{div} \mathbf{A} + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$ .

OR

b) If  $\vec{a}$  is a constant vector, prove that  $\text{curl} \frac{a \vec{r}}{r^3} = -\frac{a}{r^3} + \frac{3a}{r^5} (\mathbf{a} \cdot \vec{r})$ .

4. a) If  $\mathbf{F} = (x+y^2) \mathbf{i} - 2x \mathbf{j} + 2yz \mathbf{k}$  evaluate  $\int \mathbf{F} \cdot \mathbf{N} ds$  where s is the surface of the plane  $2x+y+2z=6$  in the first octant.

OR

b) If  $\mathbf{F} = (2x^2 - 3z) \mathbf{i} - 2xy \mathbf{j} - 4x \mathbf{k}$  evaluate i)  $\int_V \nabla \cdot \mathbf{F} dV$  and ii)  $\int_V \nabla \times \mathbf{F} dV$  where v is the closed region bounded by  $x=0, y=0, z=0, 2x+2y+z=4$ .

5. a) Verify Gauss's divergence theorem to evaluate  $\int_S (x^3 - yz)i - 2x^2 j + z k \cdot N dS$  over the surface of a cube bounded by the coordinate planes  $x=y=z=a$ .

OR

b) State and prove Stokes theorem.

### SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6). Evaluate  $\iint x^2 y dy dx$  over  $[1,2;1,3]$ .

7). Change the order of integration and evaluate  $\int_0^1 dx \int_1^x dy$ .

8). Evaluate  $\int_0^1 \int_1^2 \int_2^3 (x + y + z) dx dy dz$ .

9). Using triple integral, find the volume common to the cylinder  $x^2 + y^2 = a^2$  and  $x^2 + z^2 = a^2$

10). If  $r = a \cos t i + a \sin t j + at \tan \theta k$  find  $\left| \frac{dr}{dt} \frac{d^2 r}{dt^2} \frac{d^3 r}{dt^3} \right|$ .

11). Find  $\text{div } f$  where  $f = \text{grad } (x^3 + y^3 + z^3 - 3xyz)$ .

12). If  $A = ti - 3j + 2tk$ ;  $B = i - 2j + 2k$ ;  $C = 3i + tj - k$  find  $\int_1^2 [ABC] dt$ .

13). Find  $\int F \cdot dr$  where  $F = xy i + yz j + zx k$  over the curve  $c$  is  $r = ti + t^2 j + t^2 k$ ,  $t$  varying from -1 to 1.

14) Prove by stokes theorem  $\text{curl grad } \phi = 0$ .

15). State Gauss's divergence theorem and Green's theorem.

# GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

## Course No-14 & Course Title- ADVANCED NUMERICAL METHODS

### Course Outcomes

After successful completion of this course, the student will be able to

1. Find derivatives using various difference formulae
2. Understand the process of Numerical Integration
3. Solve Simultaneous Linear systems of Equations
4. Understand Iterative methods
5. Find Numerical Solution of Ordinary Differential Equations

### **Course Content**

#### **UNIT–I-Numerical Differentiation**

Derivatives using Newton's forward difference formula-Newton's backward difference formula- Derivatives using central difference formula -Stirling's interpolation formula -Newton's divided difference formula.

#### **UNIT– II-Numerical Integration**

General quadrature formula on errors –Trapezoidal rule–Simpson's 1/3 rule-Simpson's 3/8 rule- Weddle's rule-Euler-Maclaurin formula of summation and quadrature –The Euler transformation.

#### **UNIT– III-Solution of Simultaneous Linear systems of Equations–I**

Solution of linear systems-Direct Methods-Matrix inversion method–Gaussian elimination method- Gauss Jordan Method.

#### **UNIT– IV-Solution of Simultaneous Linear systems of Equations–II**

Method of factorization-solution of Tridiagonal systems-Iterative methods-Jacobi's method- Gauss – Siedal method.

#### **UNIT– V-Numerical Solution of Ordinary Differential Equations**

Introduction–solution of Taylor's series–Picard's method of successive approximations –Euler's method – Modified Euler's method – Runge-Kutta methods.

### Activities

Seminar/Quiz/Assignments/Applications of Numerical methods to Real life Problem  
/ Problem Solving Sessions.

### Text Book

Numerical Analysis by G.Shanker Rao, New Age International Publications

### Reference Books

1. Applied Numerical Analysis by Curtis F.Gerald and Patrick O.Wheatley, Pearson Publications.
2. Numerical Methods for Scientific and Engineering Computation by M.K. Jain, S.R.K.Iyengar and R.K.Jain, New Age International Publishers.

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**GOVERNMENT COLLEGE FOR WOMEN,(AUTONOMOUS,)SRIKAKULAM**

III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V

**TIME: 3Hrs                      COURSE-14, ADVANCED NUMERICAL METHODS**

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**Max.Marks:60**

**SECTION – A**

**Answer all five questions each question carries ten marks.**

**5x8M=40M**

1) a). The following table of values of x and y is given

|   |       |       |       |       |       |       |       |
|---|-------|-------|-------|-------|-------|-------|-------|
| X | 0     | 1     | 2     | 3     | 4     | 5     | 6     |
| Y | 6.989 | 7.403 | 7.781 | 8.129 | 8.451 | 8.750 | 9.030 |
|   | 7     | 6     | 5     | 1     | 0     | 6     | 9     |

Find  $\frac{dy}{dx}$  when  $x=1, x=3, x=6$ .

(OR)

b). From the following table, find the value of x for which y is maximum and find this value of y.

|   |        |        |        |        |        |
|---|--------|--------|--------|--------|--------|
| X | 1.2    | 1.3    | 1.4    | 1.5    | 1.6    |
| Y | 0.9320 | 0.9636 | 0.9855 | 0.9975 | 0.9996 |

2) a). Derive Newton-Cote's Quadrature Formula.

(OR)

b). Evaluate  $\int_0^1 \frac{1}{1+x} dx$  i) by Trapezoidal rule and Simpson's  $\frac{1}{3}$  rule ii) using Simpson's  $\frac{3}{8}$  rule.

3) a). Solve the following system using Gauss Jordan method  $x+y+z=6$ ,  $2y+5z=-4$ ,  $2x+5y-z=27$ .

(OR)

b). Solve the following system using matrix inversion method :  $x+y+z=3$ ;  $x+2y+3z=4$ ;  $x+4y+9z=6$ .

4) a). Solve the following system using Gauss-Seidel method:  $10x+y+z=12$ ,  $2x+10y+z=13$ ,  $2x+2y+10z=14$ .

(OR)

b). Solve the following system by the method of triangularisation:  $2x-3y+10z=3$ ;  $-x+4y+2z=20$ ;  $5x+2y+z=-$

5) a). Solve  $y'' = x - y^2$ ,  $y(0)=1$  using Taylor's series method and compute  $y(0.1)$  and  $y(0.2)$ .

(OR)

b). Obtain the value of  $y$  at  $x=0.1, 0.2$  using Runge-Kutta method of i) second order ii) third order iii) fourth order for the differential equation  $y' + y = 0$ ,  $y(0)=1$ .

### SECTION-B

**Answer any FIVE questions. Each question carries FOUR marks**

**5X4M=20M**

6). Write Newton's forward and backward difference formulas.

7). Find  $f'(0.4)$  using Newton's backward difference formula for the following table

|      |         |         |         |         |
|------|---------|---------|---------|---------|
| x    | 0.1     | 0.2     | 0.3     | 0.4     |
| f(x) | 1.10517 | 1.22140 | 1.34986 | 1.49182 |

8). Evaluate  $\int_0^1 x^3 dx$  with five sub-intervals by Trapezoidal rule.

9). State the Weddle's rule.

10). Solve the following system using Matrix inverse method  $2x+3y=13$ ,  $3x+4y=18$ .

11). Solve the following system using Gauss elimination method  $3x+y+z=1$ ,  $x+2y+3z=4$ ,  $2x+y+z=2$

12). Solve the following system using Jacobi's Method  $5x+y=6$ ,  $2x+4y=8$ .

13). Find an LU decomposition of the matrix and solve the linear system  $AX=B$ .

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

14). Using Euler's method, solve for  $y$  at  $x=2$  from  $\frac{dy}{dx} = 3x^2 + 1$ ,  $y(1)=2$  taking step size  $h=0.25$ .

15). Find the value of  $y$  for  $x=0.4$  by Picard's method, given that  $\frac{dy}{dx} = x^2 + y^2$ ,  $y(0)=0$ .

# GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

## Course No-15 & Course Title- NUMBER THEORY

### Learning Outcomes

After successful completion of the course, students will be able to

1. Understand the fundamental theorem of arithmetic
2. Understand Mobius function, Euler quotient function, The Mangoldt function, Liouville's function, The divisor functions and the generalized convolutions.
3. Understand Euler's summation formula, application to the distribution of lattice points and the applications to  $\mu(n)$  and  $\Lambda(n)$
4. Understand the concepts of congruencies, residue classes and complete residue systems.
5. Comprehend the concept of quadratic residues mod  $p$  and quadratic non-residues mod  $p$ .

### UNIT-I-The Fundamental Theorem of Arithmetic

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, The greatest common divisor of more than two numbers

### UNIT-II-Arithmetical Functions And Dirichlet Multiplication

Introduction-The Mobius function  $\mu(n)$  – The Euler quotient function  $\varphi(n)$  – A relation connecting  $\varphi$  and  $\mu$  - A product formula for  $\varphi(n)$  - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula-The Mangoldt function  $\Lambda(n)$ - multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function  $\lambda(n)$  - The divisor functions  $\sigma_\alpha(n)$

### UNIT-III-Averages Of Arithmetical Functions

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula- Some elementary asymptotic formulas-The average order of  $d(n)$ - The average order of the divisor functions  $\sigma_\alpha(n)$ - The average order of  $\varphi(n)$ - An application to the distribution of lattice points visible from the origin-The average order of  $\mu(n)$  and  $\Lambda(n)$ -The partial sums of a Dirichlet product- Applications to  $\mu(n)$  and  $\Lambda(n)$

### UNIT-IV-Congruences

Definition and basic properties of congruences- Residue classes and complete residue systems-

Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo  $p$ . Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Application of the Chinese remainder theorem

### **UNIT-V-Quadratic Residues and the Quadratic Reciprocity Law**

Quadratic Residues, Legendre's symbol and its properties, Evaluation of  $(-1/p)$  and  $(2/p)$ , Gauss lemma, The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol, Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums, Another proof of the quadratic reciprocity law.

#### Activities

Seminar/Quiz/Assignments/Applications of Number theory to Real life Problem/ Problem Solving Sessions

#### TextBook

Introduction to Analytic Number Theory by T.M.Apostol, Springer Verlag-NewYork, Heidelberg- Berlin-1976.

#### Reference Books

1. Elementary Number Theory by G.A.Jones and J.M.Jones,,Springer
2. Elementary Number Theory by David, M. Burton,2nd Edition UBS Publishers.
3. Number Theory by Hardy & Wright, Oxford Univ.,Press.
4. Elements of the Theory of Numbers by Dence, J.B&DenceT.P, Academic Press

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