

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-I

Credits-4

Hrs/Wk-5

Course No-1 & Course Title- Differential Equations

Course Objectives

1. To introduce the concepts and methods for solving first-order differential equations, including exact, linear, and Bernoulli equations.
2. To understand special types of first-order differential equations such as Clairaut's equations and those solvable for p, x or y .
3. To develop techniques for solving higher-order linear differential equations with constant coefficients.
4. To apply the operator method for finding particular integrals of non-homogeneous differential equations with various types of right-hand side functions.
5. To learn the method of variation of parameters for solving non-homogeneous differential equations.

Course Outcomes

After successful completion of the course, the student will be able to

1. Solve exact differential equations, linear equations, Bernoulli's equations, and equations reducible to exact form using integrating factors.
2. Analyze and solve first-order differential equations that are solvable for p, x and y , including Clairaut's equations.
3. Solve homogeneous and non-homogeneous linear differential equations of higher order with constant coefficients using operator methods.
4. Compute particular integrals for non-homogeneous equations when the right-hand side is a polynomial, exponential, or trigonometric function.
5. Solve non-homogeneous differential equations using the method of variation of parameters and other applicable techniques.

Unit-1-Differential Equations of first order and first degree

Linear Differential Equations – Bernoulli's Equations - Exact Differential Equations –Integrating factors - Equations reducible to Exact Equations by Integrating Factors -

i) Inspection Method $\frac{1}{Mx+Ny}$ _____

ii) $\frac{1}{Mx-Ny}$

UNIT-2-Differential Equations of first order but not of first degree

Equations solvable for p , Equations solvable for y , Equations solvable for x –Clairaut's equation

UNIT-3-Higher order linear differential equations

Solutions of homogeneous linear differential equations of order n with constant coefficients -
Solutions of non-homogeneous linear differential equations with constant coefficients by means of polynomial operators

(i) P.I. of $(D) = Q$ when $Q(x)=e^{ax}$

(ii) P.I. of $(D) = Q$ when $Q(x)=\sin bx$ or $\cos bx$

UNIT-4-Higher order linear differential equations (continued.)

Solution to a non-homogeneous linear differential equation with constant coefficients

(i) P.I. of $(D) = Q$ when $Q = bx^k$

(ii) P.I. of $(D) = Q$ when $Q = e^{ax}V$, where V is a function of x

UNIT-5-Higher order linear differential equations with non-constant coefficients

Solution to a non-homogeneous linear differential equation with constant coefficients P.I. of $(D) = Q$ when

$Q = xV$, where V is a function of x .

Linear differential Equations with non-constant coefficients; Method of variation of parameters

Additng:

Unit-1- $\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$ and $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$ methods.

Unit-2-Equations that do not contain x (or y)- Orthogonal Trajectories: Cartesian and Polar forms.

Unit-3- $f(D)y = Q$ when Q is a function of x .

Unit-5- P.I. of $f(D)y = Q$ when $Q=x^m V$, where V is a function of x -Linear differential Equations with non-constant coefficients; Cauchy-Euler Equation; Legendre Equation.

Activities

The activities planned throughout the Differential Equations course include variety of interactive and evaluative methods such as quizzes, assignments, seminars, and student presentations. Students will also engage in a mini project, prepare concept flowcharts, and participate in operator method chart activities. Peer teaching sessions, LMS-based online quizzes, and board work challenges will foster collaborative and digital learning. Additionally, poster presentations on applications and visual aids like chalk talks will be incorporated to support diverse learning styles and deepen conceptual clarity.

Text Book

Differential Equations and Their Applications by Zafar Ahsan, published by Prentice-Hall of India Pvt. Ltd, New Delhi-Second edition.

Reference Books

1. Ordinary and Partial Differential Equations by Dr. M.D. Raisinghania, published by S. Chand & Company, New Delhi.
2. Differential Equations with applications and programs – S. Balachandra Rao & HR Anuradha-Universities Press.
3. Differential Equations -Srinivas Vangala & Madhu Rajesh, published by Spectrum University Press

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM
I B.Sc. HONOURS MATHEMATICS (MAJOR)-SEMESTER-I

TIME:3Hrs.

COURSE-1, DIFFERENTIAL EQUATIONS

MARKS: 60 M

SECTION-A

Answer ALL FIVE Questions Each question carries EIGHT marks.

5 X 8M = 40M.

1) a) Solve $\frac{dy}{dx}((x^2y^3 + xy) = 1$

(OR)

b) Solve $(1+e^{x/y}) dx + e^{x/y}(1-x/y) dy = 0$

2) a) Solve $p^2 + 2p \cot x = y^2$

(OR)

b) Solve $2px = 2 \tan y + p^3 \cos^2 y$

3) a) Solve $\frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = e^{ax} + e^{bx}$

(OR)

b) Solve $(D^2+4)y = e^x + \sin 2x + \cos 2x$

4) a) Solve $(D^2-6D+13)y = 8e^{3x} \sin 2x$

(OR)

b) Solve $(D^2-4D+3)y = 2xe^{3x} + 3e^x \cos 2x$

5) a) Solve $(D^2+1)y = \operatorname{cosec} x$ by using the method of variation of parameters

(OR)

b) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$

SECTION -B

Answer any FIVE of the following .Each question carries FOUR marks.

5 X 4M = 20M

6. Solve $(x^2 + 1) \frac{dy}{dx} + 4xy = 1/x^2 + 1$

7. Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

8. Solve $y = 2xp + x^2p^4$

9. Solve $(py+x)(px-y) = 2p$

10. Solve $(D^4-4D^3+6D^2-4D+1)y = 0$

11. Solve $(D^2+4)y = \sin 2x$

12. Solve $(D^2-2D+1)y = x^2 e^{3x}$

13. Solve $(D^2-4)y = x \sinh x$

14. Solve $\frac{d^2y}{dx^2} + (x-2) \frac{dy}{dx} - 2y = x^3$

15. Solve $(x^2 D^2 + xD - 1)y = x^2 e^x$

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-I

Credits-4

Hrs/Wk-5

Course No-2 & Course Title- ANALYTICAL SOLID GEOMETRY

Course Objectives

1. To introduce fundamental concepts of planes, lines, and spheres in 3D geometry.
2. To develop analytical skills for deriving equations of planes, lines, and spheres in different forms.
3. To analyze geometric relationships, including angles, distances, and intersections between lines, planes, and spheres.
4. To study advanced properties of spheres, such as tangents, polar planes, and orthogonality conditions.
5. To apply geometric principles to solve problems involving co-planarity, shortest distances, and sphere-line/plane interactions

Course Outcomes

After completing this course, students will be able to

1. Derive and interpret equations of planes and lines in various forms.
2. Compute angles, distances, and intersection conditions between geometric elements (lines, planes, spheres).
3. Determine co-planarity of lines and solve problems involving shortest distances in 3D space.
4. Analyze sphere-related problems, including tangents, intersections, and circle equations in 3D.
5. Apply advanced concepts like polar planes, conjugate points, and orthogonality conditions of spheres.

Unit-1-The Plane

Equation of plane in terms of its intercepts on the axis - Equations of the plane through the given points - Length of the perpendicular from a given point to a given plane- Bisectors of angles between two planes - Combined equation of two planes.

Unit-2-The Line

Equation of a line in various forms- Angle between a line and a plane - The condition that a given line may lie in a given plane - The condition that two given lines are coplanar - Number of arbitrary constants in the equations of straight line - Sets of conditions which determine a line

Unit-3-The Line (continued)

The shortest distance between two lines - The length and equations of the line of shortest distance between two straight lines - Length of the perpendicular from a given point to a given line.

Unit-4-The Sphere

Definition and equation of the sphere - Equation of the sphere through four given points - Plane sections of a sphere - Intersection of two spheres - Equation of a circle - Sphere through a given circle - Intersection of a sphere and a line.

Unit-5- Spheres (continued)

Power of a point - Tangent plane - Plane of contact; Polar plane-Pole of a Plane - Conjugate points - Conjugate Planes-Angle of intersection of two spheres - Conditions for two spheres to be orthogonal - Radical plane- Coaxial system of spheres -Limiting Point.

Adding:

Unit-1- Angle between two Planes-Two sides of a Plane-Distance between parallel planes -Orthogonal projection on a plane.

Unit-3- Area of a triangle-Intersection of three planes.

Unit-5--Touching spheres -Radical line- Radical centre- Simplified form of the equation of two spheres.

Activities:

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text Book:

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand & Company Ltd. 7th Edition.

Reference Books:

1. A text Book of Analytical Geometry of Three Dimensions, by P.K.Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.
2. Co-ordinate Geometry of two and three dimensions by P.Balasubrahmanyam, K.Y.Subrahmanyam, G.R.Venkataraman published by Tata McGraw-Hill Publishers.
3. Solid Geometry by B. Rama Bhupal Reddy, published by Spectrum University Press.

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS), SRIKAKULAM

I B.Sc. HONOURS MATHEMATICS (MAJOR)-SEMESTER-I

TIME: 3Hrs COURSE-2-ANALYTICAL SOLID GEOMETRY Max.Marks:60Marks

Course No-3 & Course Title- GROUP THEORY

Course Objectives

1. To introduce students to the foundational concepts of algebraic structures with a focus on groups.
2. To develop an understanding of subgroups, cosets, and their relevance in group theory.
3. To explore the properties and significance of normal subgroups and their role in constructing quotient groups.
4. To study and apply the concepts of group homomorphisms, isomorphisms, and the fundamental theorem of homomorphism.
5. To examine the structure and properties of permutation and cyclic groups, including their role in group classification.

Course Outcomes

After successful completion of this course, the student will be able to

1. Understand the definition and basic properties of groups, including finite and infinite groups, and construct composition tables.
2. Analyze subgroups and cosets, apply Lagrange's Theorem, and understand the structure of a group through its subgroups.
3. Identify and verify normal subgroups, and understand their role in forming quotient groups.
4. Understand and apply homomorphisms and isomorphisms, including the fundamental homomorphism theorem and its applications.
5. Work with permutations, transpositions, and cyclic groups, and understand their properties and significance in group theory, including Cayley's Theorem.
- 6.

Unit – 1 Groups

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties
Finite and Infinite groups – examples – order of a group, Composition tables with examples.

Unit – 2 Sub Groups

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition-examples-criterion for a complex to be a subgroup; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – properties of Cosets – Index of a subgroups of a finite group – Lagrange’s Theorem.

Unit – 3 Normal Subgroups

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups Sub group of index 2 is a normal sub group

Unit – 4 Homomorphisms

Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – Fundamental theorem on Homomorphism and applications.

Unit – 5 Permutations and Cyclic Groups

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley’s theorem.

Cyclic Groups - Definition of cyclic group – elementary properties.

Adding:

Unit-1- Order of an element of a group

Unit-2- Normalizer of an element of a group-Self-conjugate element of a group.

Unit-3-Simple group.

Unit-5-Classification of cyclic group.

Activities:

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text Book:

Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, Krishna Prakashan Media Pvt. Ltd., Meerut.

Reference Books:

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

SECTION-A

Answer ALL FIVE Questions. Each question carries EIGHT marks.

5x8M=40M.

- 1) a) Show that the set
- Q^+
- of all positive rational numbers forms an abelian group under the

Composition defined by 'o' such that $aob = \frac{ab}{3}$ for $a, b \in Q^+$.

(OR)

- b) Prove that the set of
- n^{th}
- roots of unity under multiplication forms a finite abelian group.

- 2) a) If H and K are two subgroups of a group G then Prove that HK is a subgroup of G iff

$$HK=KH.$$

(OR)

- b) State and prove Lagrange's Theorem.

- 3) a) Prove that a subgroup H of a group G is a Normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(OR)

- b) If H is a subgroup of G and N is normal subgroup of G, then i)
- $H \cap N$
- is a normal subgroup ii)
- N
- is normal subgroup of
- HN
- .

- 4) a) State and prove fundamental theorem of Homomorphism of Groups.

(Or)

- b) If
- f
- is homomorphism of a group G, then the Kernel of
- f
- is a normal subgroup of G.

- 5) a) State and prove Cayley's Theorem.

(OR)

- b) If
- P
- is a prime number, then every group of order
- P
- is a Cyclic Group.

SECTION-B

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

- 6). Show that in a Group G, for
- $a, b \in G$
- ,
- $(ab)^2 = a^2b^2$
- iff G is abelian.

- 7). Prove that Cancellation laws hold in a group.

- 8). If
- H_1
- and
- H_2
- are two Subgroups of a Group G then
- $H_1 \cap H_2$
- is also a Subgroup of G.

- 9). Any two right cosets of a Subgroup are either disjoint or identical

- 10). Prove that a Subgroup H of a Group G is Normal, iff
- $xHx^{-1} = H, \forall x \in G$
- .

- 11). The intersection of any two normal subgroups of a group is a normal subgroup.

- 12). If
- f
- is a homomorphism from a group G into a group
- G'
- then
- $\text{Ker } f$
- is a normal subgroup of G.

- 13). The homomorphic image of a group is a group.

- 14). Examine whether the following Permutations is Even (or) Odd.
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 & \end{pmatrix}$
- .

- 15). If G is an infinite cyclic group, then G has exactly two generators.

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-I I

Credits-4

Hrs/Wk-5

Course No-4 & Course Title- ELEMENTARY REAL ANALYSIS

Course Objectives:

1. To develop a strong foundation in the real number system and its axiomatic structure.
2. To introduce the concepts of order, bounds, completeness, and related foundational properties of real numbers.
3. To explore the properties of sets in real analysis, including neighborhoods, limit points, open and closed sets.
4. To build analytical skills in handling sequences, convergence criteria, and monotonicity.
5. To understand the behavior of in finite series and apply standard convergence tests effectively.

Course Outcomes:

After successful completion of this course, the student will be able to

1. Understand the real number system, its axioms, and properties, including completeness, supremum, and infimum.
2. Apply the Archimedean property, denseness, and concepts of neighborhoods, limit points, and derived sets in problem-solving.
3. Analyze sequences for boundedness and convergence using definitions and the Cauchy criterion.
4. Understand the concept of subsequences, apply the Bolzano-Weierstrass theorem, and test convergence using Cauchy's general principle.
5. Determine the convergence of infinite series using various tests and solve related analytical problems.

Unit – 1

Real number system - Field axioms–Properties of real numbers - Order axioms – Properties of Order relation -Principle of induction - Extended real number system – Modulus of a real number – Properties of modulus– Triangle property -Aggregates – Finite and infinite aggregates – Boundedness of an aggregate –Least upper bound (supremum) and greatest lower bound (infimum) of an aggregate – Properties of boundedness– Completeness axiom – Dedekind's theorem- Theorem on Dedekind's axiom and completeness axiom.

Unit – 2

Archimedean Property- It's corollaries– Integral part of a real number –Denseness of the real number system – Intervals – Neighbourhood of a point -Limit point of an aggregate – Derived Set-Bolzano - Weierstrass theorem – Interior point of a set - Open and closed Sets – It's properties (without proofs)- Countable and uncountable sets -Properties of countable sets.

Unit –3

Sequences – Operations of sequences -Subsequences - Range and Boundedness of Sequences - Limit of a sequence and Convergent sequence – Divergent sequence – Uniqueness of a limit – Sandwich theorem on sequences-Monotone sequences - Problems

Unit – 4

Limit Point of a Sequence-Bolzano-Weierstrass theorem on subsequences–Cauchy Sequences – Cauchy's general principle of convergence – Problems

Unit – 5

Infinite Series – Convergence and divergence of series - Cauchy's general principle of convergence for series – Series of non-negative terms -Convergence of geometric series – p series test - comparison test –D'Alembert's ratio test– Cauchy's n^{th} root test – problems.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

TextBook

An Introduction to Real Analysis by Robert G.Bartle and Donald R. Sherbert, John Wiley and sons Pvt. Ltd

Reference Books

- 1.Elements of Real Analysis by Shanthi Narayan and Dr. M.D. Raisinghania, S.Chand& Company Pvt. Ltd., New Delhi.
- 2.Principles of Mathematical Analysis by Walter Rudin, McGraw-Hill Ltd.

GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS), SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTE-III

TIME: 3Hrs

COURSE-4, ELEMENTARY REAL ANALYSIS

Max.Marks:60

SECTION-A

Answer ALL FIVE Questions. Each question carries EIGHT marks.

5x8M=40M.

Course No-5 & Course Title- RING THEORY

Course Objectives

The course aims to:

1. Introduce the fundamental concepts and properties of rings, fields, and integral domains.
2. Explain the structure and significance of subrings and ideals, including prime and maximal ideals.
3. Construct quotient rings and develop composition tables for finite rings.
4. Explore ring homomorphisms, isomorphisms, and apply the fundamental theorems of ring homomorphisms.
5. Study polynomial rings, including operations, division algorithm, irreducibility, and ideal structures.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Understand and differentiate between rings, integral domains, and fields, and describe their algebraic properties.
2. Identify and construct subrings and various types of ideals, and determine when a ring qualifies as a field.
3. Analyze quotient rings, build composition tables for finite rings, and distinguish between prime and maximal ideals.
4. Applying homomorphisms and isomorphisms effectively, and interpret the fundamental homomorphism theorems.
5. Solve problems involving polynomial rings over fields, including division algorithms, factorization, and irreducibility criteria.

Course Content

Unit – 1

Definition of a Ring and Examples –Basic properties – Commutative ring - Boolean ring – Zero Divisors of a ring - Cancellation Laws – Integral Domain – Division ring – Field - Idempotent and nilpotent elements in a ring and integral domain.

Unit – 2

The Characteristic of a Ring - The characteristics of integral domain, field, Boolean ring - Definition and examples of Subrings – Necessary and sufficient condition for a nonempty subset to be a subring – Algebra of Subrings – Centre of a ring – Ideals – Algebra of ideals – A commutative ring with unity and without proper ideals is a field.

Unit – 3

Principal ideal – Principal ideal ring: definition and theorems Cosets in ring structure - Quotient ring : definition, examples and theorems– Euclidean rings : definition, examples and theorems.

Unit – 4

Homomorphism of Rings – Definition and Elementary properties – Kernel of a homomorphism – Isomorphism – Fundamental theorem of homomorphism of rings – Maximal and prime Ideals.

Unit – 5

Polynomials over a ring – Algebra of polynomials –Degree of a polynomial and related problems -- The Division Algorithm in $F[x]$ – Remainder and Factor Theorems– Irreducible Polynomials – Ideal structure in $F[x]$ – Uniqueness of Factorization in $F[x]$.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Textbook

Modern Algebra by A.R. Vasishta and A.K. Vasishta, Krishna Prakashan Media Pvt. Ltd.

Reference books

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others, Pearson Education India

SECTION-A

Answer ALL FIVE Questions. Each question carries EIGHT marks.

5x8M=40M.

1) a) Every finite integral domain is a field

(OR)

b) The characteristic of an integral domain is either a prime or zero

2) a) Let S be a non-empty subset of a ring R then S is a subring of R iff $a-b \in S$ and $ab \in S$ for all

$a, b \in S$

(OR)

b) If S_1 and S_2 are two Subrings of a Ring R then $S_1 \cap S_2$ is also a Subring of R .

3) a) Prove that Every Euclidean ring is a principal ideal ring.

(OR)

b) Prove that Every ideal of $\mathbb{Z}$ is a principal ideal

4) a) State and prove fundamental theorem of Homomorphism of Rings.

(Or)

b) If f is homomorphism of a ring R then the Kernel of R is an ideal of R .

5) a) State and prove the Division Algorithm in ring of polynomials.

(OR)

b) An element $a \in F$ where F is a field, is a zero of $f(x) \in F[x]$ iff $(x-a)$ is a factor of $f(x)$ in $F[x]$.

SECTION-B

Answer any FIVE of the following .Each question carries FOUR marks.

5x4M=20m

6. Every field is an integral domain

7. prove that every Boolean ring is abelian

8. The intersection of two subrings of a ring R is a subring.

9. A field has no proper non trivial ideals.

10. A field is a principal ideal ring

11. If U is an ideal of the ring R and $a, b \in R$ then prove that $a+U=b+U \Rightarrow a-b \in U$

12. If f is a homomorphism from a ring R into a ring R' then $\text{Ker} f$ is an ideal of R .

13. The homomorphic image of a ring is a ring.

14. If $f(x), g(x)$ are two non-zero polynomials of $F[x]$ where F is a field then $\deg(f(x).g(x)) = \deg f(x) + \deg g(x)$.

15. Find the units in $\mathbb{Z}_7[x]$.

GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM

SEMESTER-III

Credits-4

Hrs/Wk-5

Course No-6 & Course Title- ADVANCED REAL ANALYSIS

Course Objectives

This course is designed to:

1. Develop deep understanding of infinite series with non-negative terms and apply various convergence tests.
2. Introduce the concepts of limits and continuity, including their behavior at finite and infinite points.
3. Explore types of discontinuities and apply fundamental theorems related to continuous functions.
4. Understand differentiability and apply Mean Value Theorems in problem-solving.
5. Introduce Riemann integration and explore key properties and theorems of integrable functions.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply convergence tests such as P-test, Cauchy's root test, D'Alembert's ratio test, and Leibnitz test to analyze series.
2. Understand and evaluate limits of real-valued functions, including one-sided and infinite limits, and solve problems involving indeterminate forms.
3. Demonstrate knowledge of continuity, identify types of discontinuities, and apply theorems like Heine's, Borel's, and Bolzano's in analysis.
4. Understand and analyze differentiability, distinguish it from continuity, and apply Rolle's, Lagrange's, and Cauchy's Mean Value Theorems.
5. Evaluate Riemann integrals, verify conditions for integrability, and apply the Fundamental Theorem of Calculus in integration problems.

Unit – 1

Alternating Series – Leibnitz Test – Absolute and conditional convergence – Theorems and problems relating to them – Dirichlet's test – Abel's test (Problems only)

Unit – 2

Real valued Functions - Boundedness of a function - Monotone functions - Limit of a function - Algebra of limits - Sandwich theorem on limit point – Limits of some standard functions – forms – Infinite limits – Limits at infinity.

Unit – 3

Continuity and discontinuity of a function and examples - Heine's theorem- Modulus of a continuous function is a continuous function - Borel's theorem- Every continuous function is bounded - Every continuous and bounded function defined on $[a,b]$ attains its bounds -Bolzano's theorem - Bolzano's intermediate value theorem – Uniform continuity – Every continuous function on closed interval is uniformly continuous.

Unit – 4

The derivability of a function at a point and on an interval - Derivability and continuity of a function - Darboux's theorem (statement only) - Darboux's intermediate value theorem - Mean value Theorems : Rolle's theorem, Lagrange's theorem, Cauchy's Mean value theorem – Problems

Unit – 5

Riemann Integration – Upper and lower Riemann sums, and integrals - Riemann integrable functions - Necessary and sufficient condition for integrability – Continuous function on closed interval is integrable - Monotonic function on closed interval is integrable - Properties of integrable functions - Fundamental theorem of integral calculus – Problems

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

TextBook

An Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, John Wiley and sons Pvt. Ltd

ReferenceBooks

1. Elements of Real Analysis by Shanthi Narayan and Dr. M.D. Raisinghania, S.Chand& Company Pvt. Ltd., New Delhi.
2. Principles of Mathematical Analysis by Walter Rudin, McGraw-Hill Ltd.

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV

TIME: 3Hrs

COURSE-6, ADVANCED REAL ANALYSIS

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1.a) State and prove Leibnitz's test for the convergence of an alternating series. Hence, discuss the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

and determine whether the series is absolutely or conditionally convergent.

OR

b) State and prove Dirichlet's test for convergence of series. Use it to examine the convergence of $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$, $0 < x < 2\pi$.

2.a) Define the limit of a real-valued function at a point. State and prove the Algebra of Limits.

Using the limit laws, evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$.

OR

b) State and prove the Sandwich Theorem for limits. Hence evaluate $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$ and $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$.

3.a) State and prove Heine's theorem for continuity. Prove that the modulus of a continuous function is continuous.

OR

b) State and prove Bolzano's Intermediate Value Theorem. Using the theorem, show that the equation $x^3 + x - 1 = 0$ has at least one root in the interval $(0, 1)$.

4.a) State and prove Lagrange's Mean Value Theorem. Hence prove that if $f'(x) = 0$ for every x in an interval I , then f is constant on I .

OR

b) State and prove Cauchy's Mean Value Theorem. Apply it to evaluate $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$.

5.a) Define the upper and lower Riemann sums of a bounded function on $[a, b]$. State and prove the necessary and sufficient condition for Riemann integrability.

OR

b) State and prove that every continuous function on a closed interval $[a, b]$ is Riemann integrable. Hence evaluate $\int_0^1 x^2 dx$.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6. Define absolute convergence and conditional convergence. Give one example of each.

7. State Abel's test for convergence of a series. Test the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

8. Define a bounded function and a monotone function. Give suitable examples.

9. Evaluate the following limits: $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ and $\lim_{x \rightarrow \infty} \frac{3x^2+1}{x^2-2}$.

10. Define uniform continuity. Give an example of a function which is uniformly continuous on a closed interval.

11. State Borel's theorem and explain its significance for continuous functions.

12. Prove that derivability implies continuity at a point.

13. State Darboux's theorem and explain the Darboux intermediate value property of derivatives.

14. Define the upper Riemann integral and lower Riemann integral of a bounded function.

15. State the Fundamental Theorem of Integral Calculus and give its mathematical formulation.

GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS), SRIKAKULAM

SEMESTER-III

Credits-4

Hrs/Wk-5

Course No-7 & Course Title- THEORY OF MATRICES

Course Objectives:

This course aims to:

1. Develop an understanding of special matrices such as symmetric, Hermitian, orthogonal, and unitary matrices and their properties.
2. Explain the computation and properties of determinants and how they are used in matrix operations.
3. Teach methods to find the rank and inverse of matrices using elementary transformations.
4. Provide techniques to analyze and solve systems of linear equations using matrix methods.
5. Introduce the concepts of eigenvalues and eigenvectors, and highlight their significance in linear algebra and applications.

Course Outcomes:

Upon successful completion of this course, students will be able to:

1. Compute the determinant of a matrix using expansion and apply various properties for simplification.
2. Perform elementary row and column operations to simplify determinants and matrix expressions.
3. Determine the rank of a matrix using echelon and normal forms, and compute matrix inverses via transformations.
4. Analyze systems of linear equations (homogeneous and non-homogeneous) for consistency, and solve them using methods such as matrix inversion, Cramer's Rule, Gauss elimination, and LU decomposition.
5. Define and compute eigenvalues and eigenvectors of square matrices, and use characteristic equations in applications.

Course Content :

Unit I

Matrix – Algebra of Matrices – Matrix Multiplication – Transpose and trace of a matrix - Symmetric and Skew symmetric matrices - Hermitian and Skew Hermitian matrices - Orthogonal and Unitary matrices - Idempotent matrix - Nilpotent Matrix – Involutory Matrix – examples and related problems – Every square matrix can be expressed as a sum of a symmetric and a skew symmetric matrix – countable and uncountable sets - If $AB = A$ and $BA = B$, then A and countable and uncountable sets are idempotent.

Unit – 2

Determinant of a square matrix (of order 3) - Minors and Cofactors - Properties of determinants - Product of two determinants of the same order - Adjoint and Inverse of a matrix – Orthogonal matrix - Problems

Unit -3

Rank of a matrix - Elementary row and column operations on a matrix – Properties of elementary matrices – Reduction to Echelon form and Normal form - Inverse of a non-singular matrix by elementary transformations – Related problems – The rank of product of matrices cannot exceed the rank of either.

Unit -4

System of homogeneous linear equations – Conditions for trivial and non-trivial solutions – Solving methods of homogeneous linear equations system - System of non-homogeneous linear equations – Conditions for consistency and inconsistency – Matrix Inversion method – Cramer's Rule – Gauss method- LU decomposition method.

Unit - 5

Characteristic (Eigen) Values and Characteristic vectors of a square matrix - Characteristic polynomial and Characteristic equation of a matrix – Finding eigenvalues and eigen vectors of a square matrix.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text book

Matrices , by A.R.Vasishtha and A.K.Vasishtha, published by Krishna Prakashan Media (P) Ltd.

Reference books

1. S. H. Friedberg, A.L.Insel and L.E.Spence, Linear Algebra, Prentice Hall of India (P) Ltd, New Delhi, 2004.
2. R D Sharma and Umash Kumar Basic Applied Mathematics for physical sciences, pearson Education India (P) Ltd
3. Richard Bronson, Theory and problems of Matrix operations, Tata Mc Graw Hill 1989.

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV

TIME: 3Hrs

COURSE-7, THEORY OF MATRICES

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1. a) Define the following types of matrices with suitable examples:

- | | |
|---------------------|-------------------------|
| a.Symmetric matrix | b.Skew-symmetric matrix |
| c.Hermitian matrix | d.Skew-Hermitian matrix |
| e.Orthogonal matrix | f.Unitary matrix |
| g.Idempotent matrix | h.Nilpotent matrix |

OR

b) Let A and B be square matrices of the same order satisfying $AB = A$, $BA = B$. Prove that A and B are idempotent matrices. Also discuss the properties of transpose and trace of a matrix with suitable examples.

2. a) Define minor and cofactor of an element of a determinant. State and prove important properties

of determinants. Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$.

OR

b) Define the adjoint and inverse of a square matrix. Find the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix}$ using the adjoint method. Verify that $AA^{-1} = I$.

3.a) Define the rank of a matrix. Explain elementary row and column operations. Reduce the matrix

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 4 \end{pmatrix}$ to echelon form and hence find its rank.

OR

b) Explain the reduction of a matrix to normal form using elementary transformations. Find the

inverse of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ by elementary transformations.

4. a) Discuss the system of homogeneous linear equations $AX = 0$. State the conditions for trivial and non-trivial solutions. Solve the system

$$\begin{aligned}x + y + z &= 0, \\2x + 3y + z &= 0, \\x + 2y + 2z &= 0.\end{aligned}$$

OR

b) Explain the conditions for consistency and inconsistency of a system of non-homogeneous linear equations. Solve the system

$$\begin{aligned}x + y + z &= 6, \\2x + 3y + z &= 10, \\x + 2y + 3z &= 13\end{aligned}$$

by Gauss elimination method.

5.a) Define the characteristic values and characteristic vectors of a square matrix. Find the eigenvalues and corresponding eigenvectors of

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

OR

b) Define the characteristic polynomial and characteristic equation of a square matrix. Find the characteristic equation and eigenvalues of

$$A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Hence find the corresponding characteristic vectors.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6. Define idempotent, nilpotent and involutory matrices with suitable examples.

7. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find A^T , $\text{tr}(A)$, and $A + A^T$.

8. State any four important properties of determinants.

9. Find the inverse of $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ using the adjoint method.

10. Define the rank of a matrix. What is meant by the echelon form of a matrix?

11. Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$ to echelon form and find its rank.

12. State the condition for a system of homogeneous linear equations to have a non-trivial solution.

13. Solve by Cramer's Rule:

$$\begin{aligned} 2x + y &= 5, \\ x - y &= 1. \end{aligned}$$

14. Define eigenvalue and eigenvector of a square matrix.

15. Find the eigenvalues of $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

SEMESTER-IV

Credits-4

Hrs/Wk-5

Course No-8 & Course Title- LINEAR ALGEBRA

Course Objectives

1. To introduce the fundamental concepts of vector spaces, sub-spaces, and their algebraic structure.
2. To develop an understanding of basis and dimension of vector spaces and their associated theorems.
3. To explore linear transformations and their properties, including rank, nullity, and the Rank-Nullity Theorem.
4. To apply the Cayley-Hamilton Theorem to compute powers and inverses of matrices without using direct methods.
5. To understand the structure of inner product spaces and study orthogonality and related geometric properties.

Course Outcomes

After successful completion of this course, the student will be able to

1. Understand and apply the definitions and properties of vector spaces, subspaces, linear combinations, and linear span.
2. Determine the basis and dimension of vector spaces and subspaces, and apply related theorems including those on quotient spaces.
3. Define linear transformations and operators, compute rank and nullity, and apply the Rank-Nullity Theorem.
4. Use the Cayley-Hamilton Theorem to verify matrix equations and to compute matrix inverses and higher powers.
5. Understand inner product spaces, verify orthogonality, and apply key inequalities such as Schwarz's and Triangle inequalities.

Course Content

Unit –1

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - Addition and scalar multiplication of Vectors - Vector subspaces -Algebra of subspaces - Linear Sum of two subspaces -

linear combination of Vectors- Linear Span - Linear independence and Linear dependence of Vectors.

Unit-2

Basis of a Vector space –Problems on basis of a vector space - Finite Dimensional Vector spaces - Basis existence theorem – Extension and uniqueness theorems and problems on them -Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space – Theorems on dimensions

Unit –3

Linear transformations - linear operators- Properties of L.T- Sum and product of L.Ts - Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformation - Rank- Nullity Theorem.

Unit –4

Cayley Hamilton Theorem – Verification Problems – Finding inverse using Cayley Hamilton Theorem - Inner product spaces- Euclidean and Unitary spaces- Norm or length of a Vector - Problems

Unit –5

Schwartz inequality- Triangle Inequality- Parallelogram law- Orthogonal and Orthonormal sets and problems on them – Gram Schmidt orthogonalization Process (Only problems).

Activities:

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text Book :

Linear Algebra by J.N. Sharma and A.R. Vasishtha, published by Krishna Prakashan Media (P) Ltd.

Reference Books:

1. Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt. Ltd. 4th Edition, 2007
2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson education low priced edition), New Delhi.
3. Matrices by Shanti Narayana, published by S. Chand Publications

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV

TIME: 3Hrs

COURSE-8, LINEAR ALGEBRA

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1 (a) Let $V(F)$ be a vector space. Let W be a non-empty subset of V . The necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$.

(OR)

(b) If W_1 and W_2 are two subspaces of a vector space $V(F)$ then $L(W_1 \cup W_2) = W_1 + W_2$.

2. (a) Let W_1 and W_2 be two subspaces of a finite dimensional vector space. Then

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(OR)

(b) Find a basis for $\mathbb{R}^4$ write $(3,2,5,4)$ $(6,3,5,2)$ as numbers.

3. (a) Find $T(x,y,z)$ where $T: \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined by $T(1,1,1)=3$; $T(0,1,-2)=1$; $T(0,0,1)=-2$.

(OR)

(b) If $T: V_4(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ linear transformation defined by $T(a,b,c,d) = (a-b+c+d, a+2c-d, a+b+3c-3d)$ for $a,b,c,d \in \mathbb{R}$ then verify $\rho(T) + \vartheta(T) = \dim V_4(\mathbb{R})$.

4. (a) Find the characteristic roots and corresponding characteristic vectors of the matrix $A =$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

5. (a) State and prove Schwartz inequality.

(OR)

(b) Given $\{(2,1,3),(1,2,3),(1,1,1)\}$ is a basis of $\mathbb{R}^3$; construct an orthonormal basis

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

- 6) Prove that the intersection of any two subspaces W_1 and W_2 of vector space $V(F)$ is also a subspace.
- 7) If α, β, γ are linearly independent vectors of $V(R)$. Show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.
- 8) If $\alpha=(1,-1,0)$ $\beta=(2,1,3)$ find a basis for $\mathbb{R}^3$ containing α and β .
- 9) Let W_1 and W_2 be two subspaces of $\mathbb{R}^4$ given by $W_1=\{(a,b,c,d):b-2c+d=0\}$,
 $W_2=\{(a,b,c,d):a=d,b=2c\}$ find the basis and dimension of W_1 .
- 10) The mapping $T: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ is defined by $T(x,y,z)=(x-y, x-z)$ show that T is a linear transformation.
- 11) Let $U(F)$ and $V(F)$ be two vector spaces. Let $T:U(F) \rightarrow V(F)$ be a linear transformation then the range set $R(T)$ is a subspace of $V(F)$.
- 12) Prove that the square matrices A and A^T have the same characteristic values.
- 13) Find the characteristic roots of the matrix $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 4 & -1 \\ 2 & -1 & 4 \end{bmatrix}$.
- 14) State and prove Triangle inequality.
- 15) State and prove Parallelogram law

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-IV

Credits-4

Hrs/Wk-5

Course No-9 & Course Title-VECTOR CALCULUS

Course Objectives

1. To introduce the concept of vector differentiation and the use of vector operators such as gradient, divergence, and curl.
2. To extend the concept of definite integrals to functions of multiple variables through double and triple integrals.
3. To apply double and triple integrals in evaluating areas and volumes in different coordinate systems.
4. To provide knowledge on evaluating line, surface, and volume integrals using vector calculus techniques.
5. To understand and apply major vector integration theorems like Gauss's Divergence Theorem, Green's Theorem, and Stokes's Theorem.

Course Outcomes

After successful completion of this course, students will be able to

1. Understand and compute vector derivatives, including gradient, divergence, and curl, and apply vector identities in problem-solving.
2. Evaluate double integrals over various regions and use polar coordinates when appropriate.
3. Compute triple integrals in Cartesian and polar coordinates, and apply them to find volumes of solids.
4. Evaluate line, surface, and volume integrals using the appropriate vector calculus methods and geometric interpretations.
5. Apply Gauss's, Green's, and Stokes's theorem to solve different types of integrals and solve physical and geometrical problems

Course Content

Unit – 1

Vector differentiation – ordinary derivatives of vectors – Differentiability – Gradient – Divergence - Curl operators - Relations involving the operators - Problems

Unit – 2

Introduction to double integrals - Evaluation of double integrals – Properties of double integrals - Region of integration - double integration in Polar Co-ordinates – Jacobian and change of variables.

Unit – 3

Triple integral - Region of integration - Change of variables – Evaluation of triple integrals both in Cartesian and polar coordinates – Jacobian and change of variables.

Unit – 4

Line Integrals with examples - Surface Integral with examples - Volume integral with examples.

Unit – 5

Gauss theorem and applications of Gauss theorem - Green's theorem in the plane and applications of Green's theorem - Stokes's theorem and applications of Stokes theorem.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text Book

A text Book of Higher Engineering Mathematics by B.S. Grawal, Khanna Publishers, 43rd Edition

Reference Books

1. Vector Calculus by P.C. Matthews, Springer Verlag publications.
2. Vector Analysis by Murray Spiegel, Schaum Publishing Company, NewYork

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV

TIME: 3Hrs

COURSE-9, VECTOR CALCULUS

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1. a) Prove that $\text{curl} (AXB) = A \text{div} B - B \text{div} A + (B \cdot \nabla)A - (A \cdot \nabla)B$.

OR

b) If $\vec{a}$ is a constant vector, prove that $\text{curl} \frac{a\vec{x}}{r^3} = -\frac{a}{r^3} + \frac{3a}{r^5} (a \cdot \vec{r})$.

2. a) Evaluate $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$ by changing the order of integration.

OR

b) By changing into polar co-ordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the angular region between the circles $x^2 + y^2 = a^2, x^2 + y^2 = b^2$ ($b > a$).

3. a) Evaluate the integral $\iiint xy dx dy dz$ where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

OR

b) If R is region in the first octant bounded by the sphere $x^2 + y^2 + z^2 = a^2$, evaluate the integral $\iiint x + y + z dx dy dz$ by changing to spherical polar coordinates.

4. a) If $F = (x+y^2) i - 2xy j + 2yz k$ evaluate $\int F \cdot N ds$ where s is the surface of the plane $2x+y+2z=6$ in the first octant.

OR

b) If $F = (2x^2 - 3z) i - 2xy j - 4x k$ evaluate i) $\int_V \nabla \cdot F dV$ and ii) $\int_V \nabla \times F dV$ where v is the closed region bounded by $x=0, y=0, z=0, 2x+2y+z=4$.

5. a) Verify Gauss's divergence theorem to evaluate $\int_S (x^3 - yz)i - 2x^2 j + z k \cdot N dS$ over the surface of a cube bounded by the coordinate planes $x=y=z=a$.

OR

b) State and proof Stokes theorem.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6). If $r = a \cos t i + a \sin t j + at \tan \theta k$ find $\left| \frac{dr}{dt} \frac{d^2r}{dt^2} \frac{d^3r}{dt^3} \right|$.

7). Find $\text{div } f$ where $f = \text{grad } (x^3 + y^3 + z^3 - 3xyz)$.

8). Evaluate $\iint x^2 y \, dy \, dx$ over $[1, 2; 1, 3]$.

9). Change the order of integration and evaluate $\int_0^1 dx \int_1^x dy$.

10). Evaluate $\int_0^1 \int_1^2 \int_2^3 (x + y + z) dx dy dz$.

11). Using triple integral, find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$

12). If $A = ti - 3j + 2tk$; $B = i - 2j + 2k$; $C = 3i + tj - k$ find $\int_1^2 [ABC] \, dt$.

13). Find $\int F \cdot dr$ where $F = xy i + yz j + zx k$ over the curve c is $r = ti + t^2 j + t^2 k$, t varying from -1 to 1 .

14) Prove by stokes theorem $\text{curl grad } \phi = 0$.

15). State Gauss's divergence theorem and Green's theorem.

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-IV

Credits-4

Hrs/Wk-5

Course No-10 & Course Title- LINER PROGRAMMING PROBLEMS

Course Objectives:

1. To introduce the concept of convex sets and the foundational principles of linear programming.
2. To develop the ability to formulate real-life problems into linear programming models.
3. To provide graphical and algebraic techniques for solving linear programming problems.
4. To equip students with the knowledge of the simplex method, artificial variables, and special cases in LPP.
5. To explore issues such as degeneracy, alternative solutions, unboundedness, and infeasibility in linear programming problems.

Course Outcomes:

After successful completion of this course, students will be able to

1. Understand the structure of convex sets, convex combinations, and apply the fundamental theorem of linear programming to real-world problem formulation.
2. Solve LPPs graphically and represent LP problems in standard, canonical, and matrix forms.
3. Apply the simplex method to solve linear programming problems and interpret unbounded or multiple solutions.
4. Use artificial variable techniques such as the Big M-method and the two-phase method to handle constraints with equality and greater-than conditions.
5. Identify and resolve issues in LPP such as degeneracy, alternative and unbounded solutions, and apply the simplex method to solve systems of simultaneous equations.

Course Content

UNIT-I

Convex Set- Extreme points of a convex set- Convex combination- Convex hull- Convex polyhedron- Fundamental theorem of linear programming - Formulation of linear programming of problems (LPP)

UNIT-2

Graphical solution of linear programming problems- General formulation of LP problems- Standard form and matrix form of LP problems-Standard form and Canonical form of LP Problems

UNIT-3

Introduction of Simplex method - Definitions and notations - Computational procedure of simplex algorithm- Simple way for simplex computations – Unbounded and Alternative solutions

UNIT -4

Artificial variables- Big M-Method and its applications and Two-phase method- Alternative method of two-phase simplex method

UNIT - 5

Degeneracy in LPP and method to resolve degeneracy- Alternative solutions- Unbounded solutions- Non-existing feasible solutions- Solution of simultaneous equations by Simplex method.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text book

Scope as in “Operations Research” by S.D. Sharma, Kedar Nath, Ram Nath & Co- Meerut.

Reference Book

“Operation Research” by Kanthi Swarup- R.K. Gupta and Manmohan- S. Chand publications, New Delhi

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

II B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-IV

TIME: 3Hrs

COURSE-10, LINER PROGRAMMING PROBLEMS

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1.a) Define the following:

1. Convex set
- 2.Convex combination
- 3.Extreme point
- 4.Convex hull
- 5.Convex polyhedron

Prove that the convex combination of any two points of a convex set belongs to the set. Explain the Fundamental Theorem of Linear Programming.

OR

b) Explain the formulation of a Linear Programming Problem (LPP). Formulate the following problem as an LPP:

A company manufactures two products P and Q . Product P requires 2 hours of machine time and 1 hour of labour, while product Q requires 1 hour of machine time and 3 hours of labour. The available machine time is 100 hours and labour time is 90 hours. The profit per unit of P is ₹40 and that of Q is ₹30. Determine the objective function and constraints.

2.a) Explain the graphical method for solving a Linear Programming Problem. Solve graphically:

$$\text{Maximize } Z = 3x + 5y$$

subject to

$$x + y \leq 4,$$

$$2x + y \leq 5,$$

$$x \geq 0, y \geq 0.$$

OR

b) Explain the general formulation of an LPP. Convert the following LPP into standard form and matrix form:

$$\text{Maximize } Z = 3x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \leq 6,$$

$$x_1 + 2x_2 \leq 8,$$

$$x_1, x_2 \geq 0.$$

Also explain the difference between standard form and canonical form.

3. a) Explain the Simplex method and its basic terminology. Describe the computational procedure of the Simplex algorithm and solve:

$$\text{Maximize } Z = 3x_1 + 2x_2$$

subject to

$$x_1 + x_2 \leq 4,$$

$$2x_1 + x_2 \leq 5,$$

$$x_1, x_2 \geq 0.$$

OR

b) Explain the following with respect to the Simplex method:

1. Basic feasible solution

2. Basic variables

3. Non-basic variables

4. Slack variables

5. Entering variable

6. Leaving variable

Explain how unbounded and alternative optimal solutions are identified in the Simplex table.

4. a) Explain the Big-M method for solving LPPs involving artificial variables. Solve the following problem using the Big-M method:

$$\text{Minimize } Z = 3x_1 + 2x_2$$

subject to

$$x_1 + x_2 \geq 4,$$

$$x_1 + 3x_2 \geq 6,$$

$$x_1, x_2 \geq 0.$$

OR

b) Explain the Two-Phase Simplex method. Discuss the purpose of introducing artificial variables and explain the procedure of Phase-I and Phase-II. Illustrate the method with a suitable example.

5. a) What is degeneracy in a Linear Programming Problem? Explain the causes of degeneracy and the methods used to resolve it. Discuss how alternative and unbounded solutions can be identified from a Simplex table.

OR

b) Explain the following cases in Linear Programming:

1. Degenerate solution

2. Alternative optimal solution

3. Unbounded solution

4. Non-existing feasible solution

Illustrate each case with a suitable Simplex-table interpretation.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6. Define a convex set and an extreme point. Give suitable examples.

7. Define convex hull and convex polyhedron. State the Fundamental Theorem of Linear Programming.

8. Explain the standard form and canonical form of a Linear Programming Problem.

9. Solve graphically:

$$\text{Maximize } Z = 2x + 3y$$

subject to

$$x + y \leq 4,$$

$$x \leq 2,$$

$$y \leq 3,$$

$$x, y \geq 0.$$

10. Define the following terms used in the Simplex method:

- | | |
|--------------------|----------------------------|
| 1. Basic solution | 2. Basic feasible solution |
| 3. Basic variables | 4. Non-basic variables |

11. What is an unbounded solution? How can an unbounded solution be identified from a Simplex table?

12. What is an artificial variable? Why is it introduced in the Simplex method?

13. Explain the Big-M method and Two-Phase method briefly.

14. What is degeneracy in an LPP? Explain one method of resolving degeneracy.

15. Distinguish between:

1. Alternative solution
2. Unbounded solution

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS), SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

Course No-12 & Course Title- LINEARALGEBRA

Course Objectives

1. To introduce the fundamental concepts of vector spaces, sub-spaces, and their algebraic structure.
2. To develop an understanding of basis and dimension of vector spaces and their associated theorems.
3. To explore linear transformations and their properties, including rank, nullity, and the Rank-Nullity Theorem.
4. To apply the Cayley-Hamilton Theorem to compute powers and inverses of matrices without using direct methods.
5. To understand the structure of inner product spaces and study orthogonality and related geometric properties.

Course Outcomes

After successful completion of this course, the student will be able to

1. Understand and apply the definitions and properties of vector spaces, subspaces, linear combinations, and linear span.
2. Determine the basis and dimension of vector spaces and subspaces, and apply related theorems including those on quotient spaces.
3. Define linear transformations and operators, compute rank and nullity, and apply the Rank-Nullity Theorem.
4. Use the Cayley-Hamilton Theorem to verify matrix equations and to compute matrix inverses and higher powers.
5. Understand inner product spaces, verify orthogonality, and apply key inequalities such as Schwarz's and Triangle inequalities.

Course Content

UNIT– I-Vector Spaces-I

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - addition and scalar multiplication of Vectors - internal and external composition - Null space - Vector subspaces -Algebra of subspaces - Linear Sum of two subspaces - linear combination of Vectors- Linear span Linear

independence and Linear dependence of Vectors.

UNIT–II-Vector Spaces-II

Basis of Vector space-Finite dimensional Vector spaces-basis extension-co-ordinates-Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space.

UNIT–III-Linear Transformations

Linear transformations-linear operators-Properties of L.T-sum and product of L.Ts-Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformations - Rank- Nullity Theorem.

UNIT–IV-Matrices

Characteristic equation-Characteristic Values-Characteristic vectors of a square matrix-Cayley Hamilton Theorem – problems on Cayley-Hamilton Theorem.

UNIT–V-Inner product space

Inner product spaces- Euclidean and unitary spaces- Norm or length of a Vector- Schwartz inequality- Triangle Inequality- Parallelogram law- Orthogonality- Orthonormal set- Problems on Gram–Schmidt orthogonalisation process - Bessel's inequality.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes, board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and engagement.

Text Book

Linear Algebra by J.N.Sharma and A.R.Vasishtha, published by Krishna Prakashan Media (P) Ltd.

Reference Books

1. Linear Algebra by Stephen H. Friedberg et al. published by Prentice Hall of India Pvt. Ltd. 4th Edition, 2007
2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson education low priced edition), New Delhi.
3. Matrices by Shanti Narayana, published by S. Chand Publications

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V

TIME: 3Hrs

COURSE-12, LINEAR ALGEBRA

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1 (a) Let $V(F)$ be a vector space. let W be a non-empty subset of V . The necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$.

(OR)

(b) If W_1 and W_2 are two subspaces of a vector space $V(F)$ then $L(W_1 \cup W_2) = W_1 + W_2$.

2. (a) Let W_1 and W_2 be two subspaces of a finite dimensional vector space. Then

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(OR)

(b) Find a basis for $\mathbb{R}^4$ write $(3,2,5,4)(6,3,5,2)$ as numbers.

3. (a) Find $T(x,y,z)$ where $T: \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined by $T(1,1,1)=3$; $T(0,1,-2)=1$; $T(0,0,1)=-2$.

(OR)

(b) If $T: V_4(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ linear transformation defined by $T(a,b,c,d)=(a-b+c+d, a+2c-d, a+b+3c-3d)$ for $a,b,c,d \in \mathbb{R}$ then verify $\rho(T) + \vartheta(T) = \dim V_4(\mathbb{R})$.

4. (a) Find the characteristic roots and corresponding characteristic vectors of the matrix $A =$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

5. (a) State and prove Cauchy-Schwarz's inequality.

(OR)

(b) State and prove Bessel's inequality.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

- 6) Prove that the intersection of any two subspaces W_1 and W_2 of vector space $V(F)$ is also a subspace.
- 7) If α, β, γ are linearly independent vectors of $V(R)$. Show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.
- 8) If $\alpha = (1, -1, 0)$ $\beta = (2, 1, 3)$ find a basis for R^3 containing α and β .
- 9) Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a, b, c, d) : b - 2c + d = 0\}$,
 $W_2 = \{(a, b, c, d) : a = d, b = 2c\}$ find the basis and dimension of W_1 .
- 10) The mapping $T: V_3(R) \rightarrow V_2(R)$ is defined by $T(x, y, z) = (x - y, x - z)$ show that T is a linear transformation.
- 11) Let $U(F)$ and $V(F)$ be two vector spaces. Let $T: U(F) \rightarrow V(F)$ be a linear transformation then the range set $R(T)$ is a subspace of $V(F)$.
- 12) Prove that the square matrices A and A^T have the same characteristic values.
- 13) Find the characteristic roots of the matrix $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 4 & -1 \\ 2 & -1 & 4 \end{bmatrix}$.
- 14) State and prove Triangle inequality.
- 15) State and prove Parallelogram law

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

Course No-13 & Course Title-VECTOR CALCULUS

Course Objectives

1. To introduce the concept of vector differentiation and the use of vector operators such as gradient, divergence, and curl.
2. To extend the concept of definite integrals to functions of multiple variables through double and triple integrals.
3. To apply double and triple integrals in evaluating areas and volumes in different coordinate systems.
4. To provide knowledge on evaluating line, surface, and volume integrals using vector calculus techniques.
5. To understand and apply major vector integration theorems like Gauss's Divergence Theorem, Green's Theorem, and Stokes's Theorem.

Course Outcomes

After successful completion of this course, students will be able to

1. Understand and compute vector derivatives, including gradient, divergence, and curl, and apply vector identities in problem-solving.
2. Evaluate double integrals over various regions and use polar coordinates when appropriate.
3. Compute triple integrals in Cartesian and polar coordinates, and apply them to find volumes of solids.
4. Evaluate line, surface, and volume integrals using the appropriate vector calculus methods and geometric interpretations.
5. Apply Gauss's, Green's, and Stokes's theorem to solve different types of integrals and solve physical and geometrical problems

Course Content

Unit-1

Introduction –Double integrals –Evaluation of double integrals –Properties of double integrals –
Region of integration - double integration in Polar Co-ordinates –
Change of variables in double integrals –change of order of integration.

Unit–2 Multiple integrals-II

Triple integral -region of integration -change of variables -Plane areas by double
integrals - surface area by double integral –Volume as a double integral, volume as a triple
integral.

Unit–3 Vector differentiation

Vector differentiation–ordinary-derivatives of vectors – Differentiability–Gradient–Divergence- Curl
operators – Formulae involving these praetors .

Unit–4 Vector integration

Line Integrals with examples – Surface Integral with examples - Volume integral with examples.

Unit–5 Vector integration applications

Gauss theorem and applications of Gauss theorem - Green’s theorem in plane and
applications of Green’s theorem - Stokes’s theorem and applications of Stokes theorem.

Activities

The activities include quizzes, assignments, seminars, and student presentations. Additional tasks
involve mini projects, concept flowcharts, operator method charts, peer teaching, LMS-based quizzes,
board work challenges, poster presentations, and visual aids like chalk talks to enhance learning and
engagement.

Text Book

A text Book of Higher Engineering Mathematics by B.S.Gawal, Khanna Publishers, 43rd Edition

Reference Books

1. Vector Calculus by P.C. Matthews, Springer Verlag publications.
2. Vector Analysis by Murray Spiegel, Schaum Publishing Company, NewYork

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V

TIME: 3Hrs

COURSE-13, VECTOR CALCULUS

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1. a) Evaluate $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$ by changing the order of integration.

OR

b) By changing into polar co-ordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the angular region between the circles $x^2 + y^2 = a^2, x^2 + y^2 = b^2$ ($b > a$).

2. a) Evaluate the integral $\iiint xy dx dy dz$ where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

OR

b) If R is region in the first octant bounded by the sphere $x^2 + y^2 + z^2 = a^2$, evaluate the integral $\iiint x + y + z dx dy dz$ by changing to spherical polar coordinates.

3. a) Prove that $\text{curl} (A \times B) = A \text{div} B - B \text{div} A + (B \cdot \nabla) A - (A \cdot \nabla) B$.

OR

b) If $\vec{a}$ is a constant vector, prove that $\text{curl} \frac{a \vec{x}}{r^3} = -\frac{a}{r^3} + \frac{3a}{r^5} (a \cdot \vec{r})$.

4. a) If $F = (x+y^2) i - 2x j + 2yz k$ evaluate $\int F \cdot N ds$ where s is the surface of the plane $2x+y+2z=6$ in the first octant.

OR

b) If $F = (2x^2 - 3z) i - 2xy j - 4x k$ evaluate i) $\int_V \nabla \cdot F dV$ and ii) $\int_V \nabla \times F dV$ where v is the closed region bounded by $x=0, y=0, z=0, 2x+2y+z=4$.

5. a) Verify Gauss's divergence theorem to evaluate $\int_S (x^3 - yz) i - 2x^2 j + z k \cdot N dS$ over the surface of a cube bounded by the coordinate planes $x=y=z=a$.

OR

b) State and proof Stokes theorem.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6). Evaluate $\iint x^2 y \, dy \, dx$ over $[1,2;1,3]$.

7). Change the order of integration and evaluate $\int_0^1 dx \int_1^x dy$.

8). Evaluate $\int_0^1 \int_1^2 \int_2^3 (x + y + z) dx dy dz$.

9). Using triple integral, find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$

10). If $r = a \cos t \, i + a \sin t \, j + at \tan \theta \, k$ find $\left| \frac{dr}{dt} \frac{d^2 r}{dt^2} \frac{d^3 r}{dt^3} \right|$.

11). Find $\text{div } f$ where $f = \text{grad } (x^3 + y^3 + z^3 - 3xyz)$.

12). If $A = ti - 3j + 2tk$; $B = i - 2j + 2k$; $C = 3i + tj - k$ find $\int_1^2 [ABC] \, dt$.

13). Find $\int F \cdot dr$ where $F = xy \, i + yz \, j + zx \, k$ over the curve c is $r = ti + t^2 j + t^2 k$, t varying from -1 to 1 .

14) Prove by stokes theorem $\text{curl grad } \phi = 0$.

15). State Gauss's divergence theorem and Green's theorem.

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS),SRIKAKULAM

SEMESTER-V

Credits-4

Hrs/Wk-5

Course No-14 & Course Title- **ADVANCED NUMERICAL METHODS**

Course Outcomes

After successful completion of this course, the student will be able to

1. Find derivatives using various difference formulae
2. Understand the process of Numerical Integration
3. Solve Simultaneous Linear systems of Equations
4. Understand Iterative methods
5. Find Numerical Solution of Ordinary Differential Equations

Course Content

UNIT–I-Numerical Differentiation

Derivatives using Newton's forward difference formula-Newton's backward difference formula- Derivatives using central difference formula -Stirling's interpolation formula -Newton's divided difference formula.

UNIT– II-Numerical Integration

General quadrature formula on errors –Trapezoidal rule–Simpson's 1/3 rule-Simpson's 3/8 rule-Weddle's rule-Euler-Maclaurin formula of summation and quadrature –The Euler transformation.

UNIT– III-Solution of Simultaneous Linear systems of Equations–I

Solution of linear systems-Direct Methods-Matrix inversion method–Gaussian elimination method-Gauss Jordan Method.

UNIT– IV-Solution of Simultaneous Linear systems of Equations–II

Method of factorization-solution of Tridiagonal systems-Iterative methods-Jacobi's method- Gauss – Siedal method.

UNIT– V-Numerical Solution of Ordinary Differential Equations

Introduction–solution of Taylor's series–Picard's method of successive approximations –Euler's method – Modified Euler's method – Runge-Kutta methods.

Activities

Seminar/Quiz/Assignments/Applications of Numerical methods to Real life Problem
/ Problem Solving Sessions.

Text Book

Numerical Analysis by G.Shanker Rao, New Age International Publications

Reference Books

1. Applied Numerical Analysis by Curtis F.Gerald and Patrick O.Wheatley, Pearson Publications.
2. Numerical Methods for Scientific and Engineering Computation by M.K. Jain, S.R.K.Iyengar and R.K.Jain, New Age International Publishers.

GOVERNMENT COLLEGE FOR WOMEN, (AUTONOMOUS,) SRIKAKULAM

III B.Sc. HONOURS MATHEMATICS(MAJOR), SEMESTER-V

TIME: 3Hrs

COURSE-14, ADVANCED NUMERICAL METHODS

Max.Marks:60

SECTION – A

Answer all five questions each question carries ten marks.

5x8M=40M

1) a).The following table of values of x and y is given

X	0	1	2	3	4	5	6
Y	6.989	7.403	7.781	8.129	8.451	8.750	9.030
	7	6	5	1	0	6	9

Find $\frac{dy}{dx}$ when $x=1, x=3, x=6$.

(OR)

b).From the following table, find the value of x for which y is maximum and find this value of y.

X	1.2	1.3	1.4	1.5	1.6
Y	0.932	0.963	0.985	0.997	0.999
	0	6	5	5	6

2) a). Derive Newton-Cote's Quadrature Formula.

(OR)

b). Evaluate $\int_0^1 \frac{1}{1+x} dx$ i) by Trapezoidal rule and Simpson's $\frac{1}{3}$ rule ii) using Simpson's $\frac{3}{8}$ rule.

3) a). Solve the following system using Gauss Jordan method $x+y+z=6, 2y+5z=-4, 2x+5y-z=27$.

(OR)

b). Solve the following system using matrix inversion method : $x+y+z=3; x+2y+3z=4; x+4y+9z=6$.

4) a). Solve the following system using Gauss-Seidel method: $10x+y+z=12, 2x+10y+z=13, 2x+2y+10z=14$.

(OR)

b). Solve the following system by the method of triangularisation: $2x-3y+10z=3$; $-x+4y+2z=20$;
 $5x+2y+z=-12$

5) a). Solve $y'' = x - y^2, y(0)=1$ using Taylor's series method and compute $y(0.1)$ and $y(0.2)$.

(OR)

b). Obtain the value of y at $x=0.1, 0.2$ using Runge-Kutta method of i) second order ii) third order iii) fourth order for the differential equation $y' + y = 0, y(0)=1$.

SECTION-B

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6). Write Newton's forward and backward difference formulas.

7). Find $f'(0.4)$ using Newton's backward difference formula for the following table

x	0.1	0.2	0.3	0.4
f(x)	1.1051	1.2214	1.3498	1.4918
	7	0	6	2

8). Evaluate $\int_0^1 x^3 dx$ with five sub-intervals by Trapezoidal rule.

9). State the Weddle's rule.

10). Solve the following system using Matrix inverse method $2x+3y=13$, $3x+4y=18$.

11). Solve the following system using Gauss elimination method $3x+y+z=1$, $x+2y+3z=4$, $2x+y+z=2$

12). Solve the following system using Jacobi's Method $5x+y=6$, $2x+4y=8$.

13). Explain Gauss – Siedal method.

14). Using Euler's method, solve for y at $x=2$ from $\frac{dy}{dx} = 3x^2 + 1, y(1)=2$ taking step size $h=0.25$.

15). Explain Picard's method of successive approximations

Course No-15 & Course Title- NUMBER THEORY

Learning Outcomes

After successful completion of the course, students will be able to

1. Understand the fundamental theorem of arithmetic
2. Understand Mobius function, Euler quotient function, The Mangoldt function, Liouville's function, The divisor functions and the generalized convolutions.
3. Understand Euler's summation formula, application to the distribution of lattice points and the applications to $\mu(n)$ and $\Lambda(n)$
4. Understand the concepts of congruencies, residue classes and complete residues systems.
5. Comprehend the concept of quadratic residues mod p and quadratic non-residues mod p .

UNIT-I-The Fundamental Theorem of Arithmetic

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, The greatest common divisor of more than two numbers

UNIT-II-Arithmetical Functions And Dirichlet Multiplication

Introduction-The Mobius function $\mu(n)$ – The Euler quotient function $\varphi(n)$ – A relation connecting φ and μ - A product formula for $\varphi(n)$ - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula-The Mangoldt function $\Lambda(n)$ - multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function $\lambda(n)$ - The divisor functions $\sigma_\alpha(n)$

UNIT-III-Averages Of Arithmetical Functions

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula- Some elementary asymptotic formulas-The average order of $d(n)$ - The average order of the divisor functions $\sigma_\alpha(n)$ - The average order of $\varphi(n)$ - An application to the distribution of lattice points visible from the origin-The average order of $\mu(n)$ and $\Lambda(n)$ -The partial sums of a Dirichlet product- Applications to $\mu(n)$ and $\Lambda(n)$

UNIT-IV-Congruences

Definition and basic properties of congruences- Residue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p . Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem

UNIT-V-Quadratic Residues and the Quadratic Reciprocity Law

Quadratic Residues, Legendre's symbol and its properties, Evaluation of $(-1/p)$ and $(2/p)$, Gauss lemma, The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol, Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums, Another proof of the quadratic reciprocity law.

Activities

Seminar/Quiz/Assignments/Applications of Number theory to Real life Problem/ Problem Solving Sessions

TextBook

Introduction to Analytic Number Theory by T.M.Apostol, Springer Verlag-NewYork, Heidelberg-Berlin-1976.

Reference Books

1. Elementary Number Theory by G.A.Jones and J.M.Jones,,Springer
2. Elementary Number Theory by David, M. Burton,2nd Edition UBS Publishers.
3. Number Theory by Hardy & Wright, Oxford Univ.,Press.
4. Elements of the Theory of Numbers by Dence, J.B&DenceT.P, Academic Press

Government College for Women (A), Srikakulam

Department of Mathematics

Semester End Examinations

Year:III

Semester : V

COURSE-NUMBER THEORY

Time:3Hrs

Max.Marks:60M

SECTION-A

Answer ALL questions. Each question carries EIGHT marks

5X8M=40M

1.a) State and prove Fundamental Theorem of Arithmetic.

(OR)

b). Find greatest common divisor d of $a = 233$, $b = 89$ and $c = 144$ and express d as linear combination of a,b,c with integer coefficients.

2.a) Show that $\varphi(mn) = \varphi(m)\varphi(n) d / \varphi(d)$, where d is greatest common divisor of m and n.

(OR)

b). If $n \geq 1$ then prove that $\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.

3.a) For any $x > 1$ then prove that $\sum_{n \leq x} \varphi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$.

(OR)

b). State and prove Euler's Summation Formula.

4.a) If then show that $a \equiv b \pmod{m}$ and $c_1 \equiv c_2 \pmod{m}$, then show that

(i) $ax + ay \equiv bx + cy \pmod{m}$ for all integers x and y.

(ii) $a^p \equiv b^p \pmod{m}$.

(OR)

b). If $(a,b) = d$ then prove that there exists integers x,y such that $ax+by = d$.

5.a) Find the legendre symbol $\left(\frac{219}{383}\right)$.

(OR)

b). State and prove Euler's Criterion Theorem for quadratic residues modulo p.

SECTION-A

Answer any FIVE questions. Each question carries FOUR marks

5X4M=20M

6. State and prove Euclid Lemma.
7. Find greatest common divisor d of $a = 1597$, $b = 2584$ and $c = 4181$.
8. Define Euler totient function $\phi(n)$ and find $\phi(100)$ and $\phi(200)$.
9. Define Liouville's function $\lambda(n)$ and find $\lambda(n)$ for $n = 1, 2, 3, 4, 5$ and $n = 6$.
10. If $x \geq 1, s > 0, s \neq 1$ then show that $\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + O(x^{-s})$ where $\zeta(s)$ is Riemann zeta function.
11. Prove that for all $x \geq 1$ then $\sum_{n \leq x} \sigma(n) = \frac{1}{12} \pi^2 x^2$.
12. Solve the linear congruence $10x \equiv 15 \pmod{45}$.
13. Using congruences prove that an integer is divisible by nine if sum of its digits is divisible by nine.
14. Define the Jacobi symbol.
15. Determine all quadratic residues modulo 7.